Palindromic n^{th} Power Sums

Ann Marie Murray
amurray@nebrwesleyan.edu
Advisor: Dr. Austin Mohr
Nebraska Wesleyan University, Lincoln, NE 68504

NEBRASKA WESLEYAN UNIVERSITY

Abstract

$434 = 11^2 + 12^2 + 13^2$

We present an algorithm implemented in Sage that lists the palindromic numbers that can be written as the sum of consecutive n^{th} powers of positive integers. This problem is a generalized version of the 125^{th} Project Euler problem.

Bounds and Initial Terms

It appears palindromic sums may be arbitrarily large, but they become increasingly difficult to find. Interestingly, the first term of the known sequences with this property do not increase with n. One may also note, referencing the table, the first terms are relatively small (three digits or less). Therefore, it seems finding the first terms of sequences with higher n^{th} powers should be simple. However, a first term where $6 \le n \le 30$ has not been found up to 10^{30} with this algorithm, but there is no obvious reason why a palindrome with this property does not exist for those greater values of n.

Further Research

Is there a palindromic sum for every n? Are there infinitely-many palindromic sums for fixed n? How can the search space be reduced?

n	Sequences
2	5, 55, 77, 181, 313, 434, 505, 545, 595, 636, 818, 1001, 1111, 1441, 1771, 4334, 6446, 17371, 17871, 19691, 21712, 41214, 42924, 44444, 46564, 51015, 65756, 81818, 97679, 99199, 108801, 127721, 137731…
3	9, 99, 9009, 14841, 76167, 108801, 239932, 828828, 886688, 2112112, 4663664, 7152517, 17333371, 17511571, 42844824, 61200216, 135666531, 658808856, 6953443596, 6961551696
4	353, 979, 16561, 998899, 2138312, 85045192129154058
5	33, 10901, 1002001

Palindromes that are the sum of nth powers of consecutive integers. Eligible sequences submitted to OEIS.

Algorithm Description

First we sum a sequence of positive integers beginning with 1 ($1^n + 2^n + 3^n + 4^n + \ldots$) until this sum exceeds some arbitrary maximum, looking for palindromes at each addition. Similarly, we start at 2 ($2^n + 3^n + 4^n + 5^n + \ldots$) checking for palindromes until the maximum has been exceeded. We continue in this manner to find all the palindromic numbers less then our arbitrary maximum which can be written as the sum of n^{th} powers of consecutive integers.

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Sage Algorithm

```
def palindromic_power_sum(n,x_max):
  success = set()
  for x_min in xrange(1,x_max^(1/n)):
    sum_powers = x_min^n
    for i in xrange(x_min+1,x_max^(1/n)+1):
      sum_powers += (i^n)
      if sum_powers > x_max:
        break
      if str(sum_powers) == str(sum_powers)[::-1]:
        success.add(sum_powers)
  return len(success), sorted(success)
```

Example

n = 3

maximum = 100

$1^3 + 2^3 = 9$

$1^3 + 2^3 + 3^3 = 36$

$1^3 + 2^3 + 3^3 + 4^3 = 100$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$

$2^3 + 3^3 = 35$

$2^3 + 3^3 + 4^3 = 99$

$2^3 + 3^3 + 4^3 + 5^3 = 224$

$3^3 + 4^3 = 91$

$3^3 + 4^3 + 5^3 = 216$