

Austin Mohr
Math 730
Homework 1

In the following problems, let Λ be an indexing set and let A and B_λ for $\lambda \in \Lambda$ be arbitrary sets.

Problem 1B1

$$\text{Show } A - \left(\bigcap_{\lambda \in \Lambda} B_\lambda \right) = \bigcup_{\lambda \in \Lambda} (A - B_\lambda).$$

Proof.

$$\begin{aligned} x \in A - \left(\bigcap_{\lambda \in \Lambda} B_\lambda \right) &\Leftrightarrow x \in A \text{ and } x \notin \bigcap_{\lambda \in \Lambda} B_\lambda \\ &\Leftrightarrow x \in A \text{ and } x \notin B_{\lambda_0} \text{ for some } \lambda_0 \in \Lambda \\ &\Leftrightarrow x \in A - B_{\lambda_0} \\ &\Leftrightarrow x \in \bigcup_{\lambda \in \Lambda} (A - B_\lambda) \end{aligned}$$

□

Problem 1H1

$$\text{Show } f^{-1} \left(\bigcup_{\lambda \in \Lambda} B_\lambda \right) = \bigcup_{\lambda \in \Lambda} f^{-1} B_\lambda.$$

Proof.

$$\begin{aligned} y \in f^{-1} \left(\bigcup_{\lambda \in \Lambda} B_\lambda \right) &\Leftrightarrow f(x) = y \text{ for some } x \in \bigcup_{\lambda \in \Lambda} B_\lambda \\ &\Leftrightarrow \exists \lambda_0 \in \Lambda \text{ such that } f(x) = y \text{ with } x \in B_{\lambda_0} \\ &\Leftrightarrow y \in f^{-1} B_{\lambda_0} \\ &\Leftrightarrow y \in \bigcup_{\lambda \in \Lambda} f^{-1} B_\lambda \end{aligned}$$

□