Classification and Generation of Spanning Trees by Isomorphism

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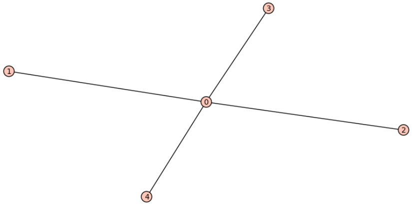
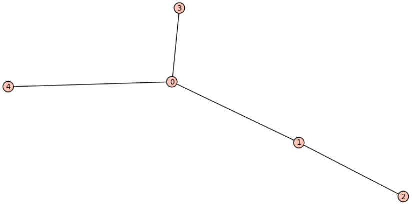
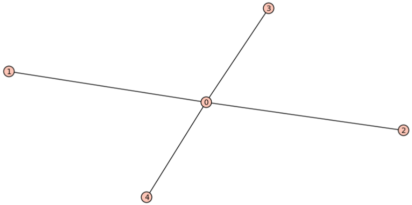
Two Spanning Trees

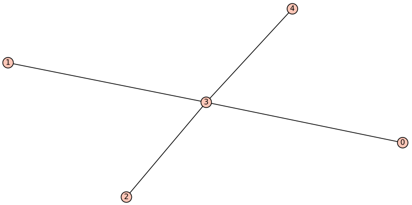




Isomorphisms

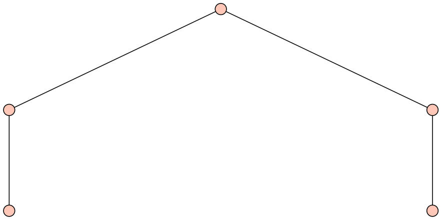
Two More Trees





Rooting and Primary Representation of the Trees

New way to represent the tree



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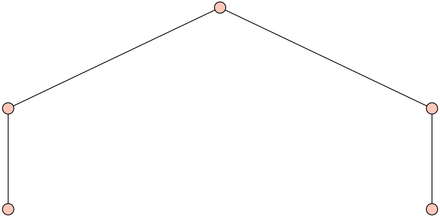

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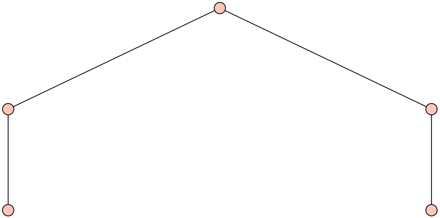

$[0, 1$

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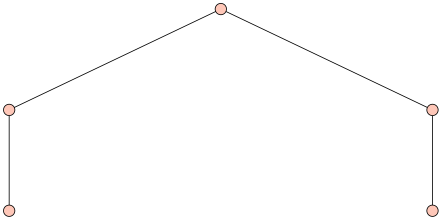

$[0, 1, 2$

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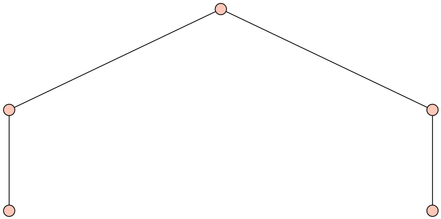

$[0, 1, 2, 1$

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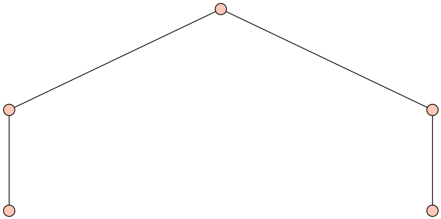

$$[0, 1, 2, 1, 2]$$

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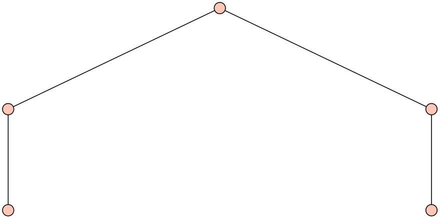

$$[0, 1, 2, 1, 2]$$

- Goal is to find each lexicographically greatest integer sequence

The Succession Algorithm

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- The successor function $s(L)$ is given by

$$s_i = \begin{cases} L_i & 0 \leq i < p \\ L_{i-(p-q)} & p \leq i \leq n-1 \end{cases}$$

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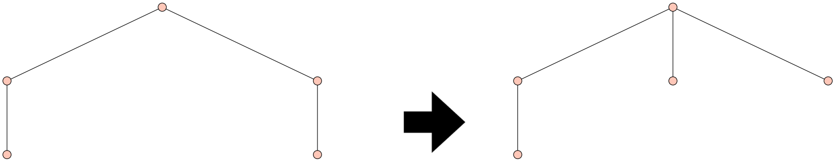
Succession Algorithm and Failure Detection

Result



[0,1,2,1,2] → [0,1,2,1,1]

Succession Algorithm and Failure Detection

- There are cases where the succession algorithm fails
- $L = [0, 1, 2, 2..., 2]$
- Starting with the path guarantees there will always be a second 1 in the integer string
- If a different failure condition is met, a different set of operations on the string guarantee a new non-isomorphic tree to be generated

Burnside's Lemma

Lemma

The number of distinct labelings for a tree T on n vertices is given by

$$\frac{n!}{|Aut(T)|}$$

Cayley's Theorem

Theorem

The total number of labeled spanning trees in a complete graph on n vertices is given by

$$n^{n-2}$$

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- Cayley's gives the total number of labeled trees on n vertices
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- The sum of all labeled trees in each isomorphic class should yield the number given by Cayley's theorem

Example

http://austinmohr.com/home/?page_id=1422

Future Work

- Subgraph Isomorphism Problem
- Number of bipartite graphs in polynomial time

References

- McKay, Brendan D., et al. "Constant Time Generation of Free Trees." SIAM Journal on Computing 15 (1986): 540-48. Print.

Thanks for listening!