University of South Carolina
Math 574: Discrete Mathematics I
Section 001
Summer I 2012

Midterm Exam

Write only your name on this sheet. Put all your work and answers on the blank paper provided.

Part A: Each question is worth **four points**. Show your work whenever appropriate to ensure you get at least partial credit, but you do not need to provide justification. (For example, you may use modus ponens without specifically mentioning it.)

1. Write the truth table for: $\sim p \vee q \to \sim q$
2. Simplify using logical equivalences: $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$
3. Write the negation of: $\forall x$, if $P(x) \vee Q(x)$, then $\sim R(x)$.
4. You are stacking red and black poker chips under the restriction that red chips always appear in groups of at least two, if they appear at all. (For example, **RR**B and BB are valid stacks, but B**R**BB is not.) Write a recurrence relation for a_n, the number of valid stacks using n chips, and give the necessary initial condition(s). (Hint: What possibilities are there to begin a stack?)

Part B: Each question is worth **six points**: four points for content and two points for form. To receive the form points, make sure your proof is neatly organized and clearly presented using complete sentences (though you may use common shorthand such as "$n \in \mathbb{Z}$").

1. Prove directly: If a is any odd integer and b is any even integer, then $a^2 + b^2$ is odd.
2. Prove by contradiction: If a and b are rational numbers with $b \neq 0$ and x is an irrational number, then $a + bx$ is irrational.
3. Prove by contraposition: For all integers n, if n^2 is not divisible by 9, then n is not divisible by 3.
4. Prove by induction: For all integers $n \geq 1$,

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

(Hint: Factor at every opportunity.)