

University of South Carolina  
Math 574: Discrete Mathematics I  
Section 001  
Summer I 2012

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Homework Set 12

Pre-Class Homework Due: 6-19

Post-Class Homework Due: 6-26

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Graphs: Definitions and Basic Properties

Before Class

- Read from the beginning of the section through Example 1.
- Skim the section “Examples of Graphs” just to see what kinds of things graphs can model. (We will look at some more during class.)

After Class

- How many edges does the complete graph on  $n$  vertices have? (Hint: Combinations)
  - How many edges does the complete bipartite  $K_{m,n}$  have? (Hint: Multiplication principle)
  - Prove every graph has at least two vertices of the same degree. (Hint : Consider separately the cases when the graph has an isolated vertex and when it has no isolated vertex. In each case, what is the smallest and largest possible degree of a vertex? Finally, try to apply the pigeonhole principle.)
  - Prove, for every integer  $n \geq 5$ , there is a graph on  $n$  vertices all having degree four. (Hint: Havel-Hakimi might give it to you, but it will be technical and boring. Instead, use induction on  $n$ . When going from  $n$  to  $n + 1$ , how can you add in a new vertex while keeping all degrees at four?)
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Trails, Paths, and Circuits

Before Class

- Nothing for today.

After Class

- # 32, 33, 34, 35 (For each question, just give one example, not two.)