University of South Carolina
Math 574: Discrete Mathematics I
Section 001
Summer I 2012

Final Exam

Write only your name on this sheet. Put all your work and answers on the blank paper provided.

Part A: Each question is worth **four points**. Show your work whenever appropriate to ensure you get at least partial credit, but you do not need to provide justification. (For example, you may use the addition rule without specifically mentioning it.)

1. Prove the following set identity algebraically (that is, without referring to elements):

$$A - (A \cap B) = A - B.$$

2. You are dealt five cards from a standard poker deck. In how many ways can you receive a full house (a three-of-a-kind and a pair simultaneously)?

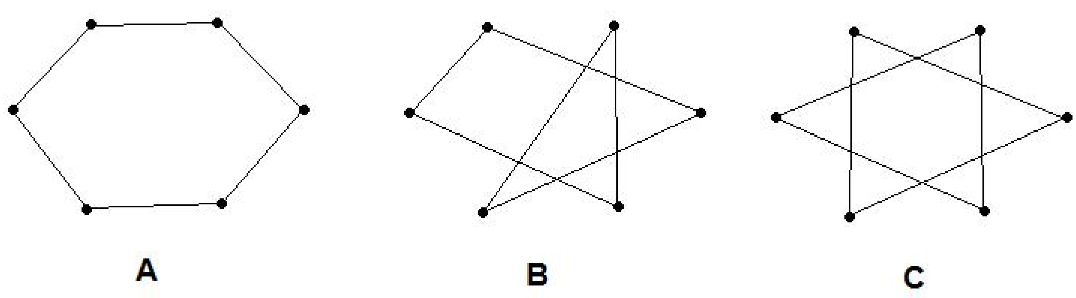
3. Using the letters A, B, C, D, E, and F at most once each, how many words of length four begin with A or end with F? (Notice some words begin with A *and* end with F.)

4. Define the function $f : \mathcal{P}(\{a, b, c\}) \to \mathbb{Z}$ by letting $f(A) = |A|$ for any $A \in \mathcal{P}(\{a, b, c\})$. (Recall $\mathcal{P}(\{a, b, c\})$ is the collection of all subsets of $\{a, b, c\}$.)

   (a) Give a specific example showing f is not one-to-one.

   (b) Give a specific example showing f is not onto.

5. In the next two questions, refer to the following graphs.



   (a) Show graphs A and B are isomorphic.

   (b) Show graphs A and C are not isomorphic.

Part B: Each question is worth **six points**: four points for content and two points for form. To receive the form points, make sure your proof is neatly organized and clearly presented using complete sentences (though you may use common shorthand such as "$n \in \mathbb{Z}$").

1. Let
$$A = \{a \in \mathbb{Z} \mid a = 6m - 5 \text{ for some } m \in \mathbb{Z}\}$$
and
$$B = \{b \in \mathbb{Z} \mid b = 3n + 1 \text{ for some } n \in \mathbb{Z}\}.$$
Prove A is a subset of B.

2. Prove the following function is one-to-one.
$$f : \mathbb{R} - \{0\} \to \mathbb{R}$$
$$f(x) = \frac{x+1}{x}$$

3. Prove, for all $n \geq r \geq 1$,
$$S_{n,r} = S_{n-1,r-1} + rS_{n-1,r},$$
where $S_{n,r}$ denotes the Stirling number of the second kind (i.e. the number of partitions of an n-element set into exactly r nonempty subsets). (Hint: There are two kinds of partitions: those in which the element n appears by itself and those in which it appears together with other elements.)

4. Prove by induction on n: For any $n \geq 2$, there is a 3-regular (i.e. all vertices have degree 3) simple graph on $2n$ vertices. (Hint: For the induction step, start with a 3-regular simple graph on $2k$ vertices and show how to build a 3-regular simple graph on $2(k+1)$ vertices from it. The fact that every 3-regular graph has two disjoint edges will be helpful.)