**University of South Carolina**
**Math 222: Math for Elementary Educators II**
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**Section 002**
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## Quiz 4-like Solutions

**1.** Find the equation of the line through (-3, -4) and perpendicular to
**a.** $y = 3x + 2$

**Solution:** The slope of the perpendicular line is the negative reciprocal of the slope of the original line. In this case, that means the slope of the perpendicular line is $-\frac{1}{3}$, so our equation looks like $y = -\frac{1}{3}x + b$. We have a point that is supposed to satisfy this equation, so we can plug it in to solve for $b$.

$$\begin{aligned} y &= -\frac{1}{3}x + b \\ -4 &= -\frac{1}{3}(-3) + b \\ -4 &= 1 + b \\ -5 &= b \end{aligned}$$

So, the equation of the perpendicular line is $y = -\frac{1}{3}x - 5$.

**b.** $y = -3x + 4$

**Solution:** This is exactly like part a, but with different numbers. The slope of the perpendicular line is $\frac{1}{3}$, so the equation looks like $y = \frac{1}{3}x + b$. Plugging in the point we know about, we can find $b$.

$$\begin{aligned} y &= \frac{1}{3}x + b \\ -4 &= \frac{1}{3}(-3) + b \\ -4 &= -1 + b \\ -3 &= b \end{aligned}$$

So, the equation of the perpendicular line is $y = \frac{1}{3}x - 3$.

**c.** $x = 2$

**Solution:** The line $x = 2$ is a vertical line (it's the line containing all the points where $x$ is 2), so the line perpendicular to it will be horizontal. Horizontal lines are of the form $y = b$ (since the slope of a horizontal line is 0). As before, we can plug in a point to find $b$.

$$y = b$$
$$-4 = b$$

Notice there was no $x$ in the equation, but that's fine. We don't have anywhere to plug the $x$-coordinate in, so we just don't plug it in. All told, we see that the equation of the perpendicular line is $y = -4$.

**d.** $y = -6$

**Solution:** The line $y = -6$ is a horizontal line (it's the line containing all the points where $y$ is -6), so the line perpendicular to it will be vertical. Vertical lines are of the form $x = c$ (since the slope of a vertical line is undefined). As before, we can plug in a point to find $c$.

$$x = c$$
$$-3 = c$$

Notice there was no $y$ in the equation, but that's fine. We don't have anywhere to plug the $y$-coordinate in, so we just don't plug it in. All told, we see that the equation of the perpendicular line is $x = -3$.

**2.** For parts a and b, draw a picture of the translation.

**2a.** Let $A$ be the point $(2, 3)$. Apply the translation $(x, y) \to (x + 1, y - 4)$ to $A$. Call the resuling point $A'$.

**Solution:** The translation says to add 1 to the $x$-coordinate and subtract 4 from the $y$-coordinate. So, $A'$ is the point (3, -1).

**2b.** Apply the translation $(x, y) \to (x - 7, y + 1)$ to $A'$. Call the resulting point $A''$.

**Solution:** The translation says to subtract 7 from the $x$-coordinate and add 1 to the $y$-coordinate. So, $A''$ is the point (-4, 0).

**2c.** Describe a single translation that sends $A$ to $A''$ directly.

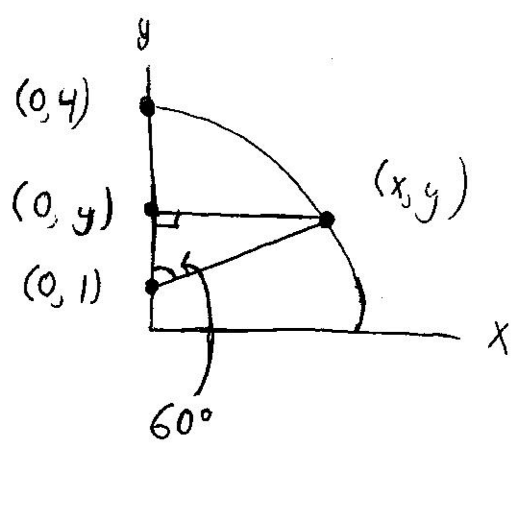
**Solution:** We want to get from the point $A$, which is (2, 3), to the point $A''$, which is (-4, 0). To do this with just one translation, we should subtract 6 from the $x$-coordinate and subtract 3 from the $y$-coordinate. So, the translation we want is $(x, y) \to (x - 6, y - 3)$.

**2d.** Guess a formula that combines two translations $(x, y) \to (x + h_1, y + k_1)$ and $(x, y) \to (x + h_2, y + k_2)$ into a single equivalent translation.

**Solution:** Think about just the $x$-coordinate first. In parts a and b, we added 1 and then subtracted 7, which gives a total change of subtracting 6. For the $y$-coordinate, we subtracted 4 and then added 1, giving a total change of subtracting 3. In both cases, we simply added the values of the translations together. So, a formula for combining two arbitrary translations would look like $(x, y) \to (x + h_1 + h_2, y + k_1 + k_2)$.

**3.** Find the coordinates of the image of the point (0, 4) after rotation 60° clockwise about the point (0, 1).

**Solution:** The rotation will give a picture something like this.



In the picture, $(x, y)$ is the image of (0, 4) after rotation. We know the distance from $(0, 1)$ to $(x, y)$ is the same as the distance from $(0, 1)$ to $(0, 4)$ (since both $(0, 4)$ and $(x, y)$ lie on the circle defined by the rotation). So, the distance from $(0, 1)$ to $(x, y)$ is 3. Notice that this distance is precisely the hypotenuse of the triangle in our picture. Since we know the hypotenuse and one angle, we can use trigonometry to find the other two sides. Let's call the vertical leg $a$ and the horizontal leg $b$.

$$\begin{aligned} \cos(60°) &= \frac{a}{3} \\ .5 &= \frac{a}{3} \\ 1.5 &= a \end{aligned}$$

So, the length of the vertical leg is 1.5, which means $y$ is 2.5 (we have to add 1 since the leg starts at (0, 1)). Now, for $b$,

$$\begin{aligned} \sin(60°) &= \frac{b}{3} \\ .87 &= \frac{b}{3} \\ 2.6 &= b \end{aligned}$$

So, the length of the horizontal leg is 2.6, which means $x$ is 2.6.

Finally, the image of (0, 4) after the rotation is (2.5, 2.6).

**4.** Find the equation of image of the line $y = 3x + 2$ after reflection over
**4a.** the $x$-axis.

**Solution:** Since we are reflecting over the $x$-axis, the $x$-intercept is a convenient point to pick because it won't move at all after the reflection. The $x$-intercept is the place where the line crosses the $x$-axis. In other words, the place where $y = 0$. We can plug in 0 for $y$ to find the appropriate $x$.

$$\begin{aligned} 0 &= 3x + 2 \\ -2 &= 3x \\ -\frac{2}{3} &= x \end{aligned}$$

So, the $x$-intercept is $(-\frac{2}{3}, 0)$.

Another easy point to find is the $y$-intercept. It's found the same way, except now $x$ is 0 and we find $y$.

$$y = 3(0) + 2$$
$$y = 2$$

So, the $y$-intercept is $(0, 2)$.

Now, reflecting over the $x$-axis leaves the $x$-coordinate of a point unchanged and reverses the sign of the $y$-coordinate (a quick sketch will help convince you of this). So, the images of our two points are $(-\frac{2}{3}, 0)$ and $(0, -2)$, respectively.

The last step is to find the equation of the line through these two points. We already know the $y$-intercept; it's the point $(0, -2)$. We can use the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 0}{0 - (-\frac{2}{3})} \\ &= \frac{-2}{\frac{2}{3}} \\ &= -3 \end{aligned}$$

All told, the equation of the line after reflection is $y = -3x - 2$. Notice the slope is just the negative of the original slope, which you might have suspected from a picture. This is one of the great things about mathematics; your intuition gives you a plausible hypothesis ("I think the slope of the reflected line will be the negative of the original slope.") and the machinary of algebra confirms your suspiscion.

**4b.** the $y$-axis.

**Solution:** We'll start with the same two points as in part a (they were $(-\frac{2}{3}, 0)$ and $(0, 2)$).

Reflecting over the $y$-axis leaves the $y$-coordinate of a point unchanged and reverses the sign of the $x$-coordinate (a quick sketch will help convince you of this). So, the images of our two points are $(\frac{2}{3}, 0)$ and $(0, 2)$, respectively.

The last step is to find the equation of the line through these two points. We already know the $y$-intercept; it's the point $(0,2)$. We can use the slope formula to find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2-0}{0-\frac{2}{3}} \\ &= \frac{2}{-\frac{2}{3}} \\ &= -3 \end{aligned}$$

All told, the equation of the line after reflection is $y = -3x + 2$. Again, the slope is just the negative of the original slope, which you might have suspected from a picture.

**4c.** the line $y = x$.

**Solution:** We'll start with the same two points as in part a (they were $(-\frac{2}{3}, 0)$ and $(0,2)$).

Reflecting over the line $y = x$ exchanges the $x$-coordinate and $y$-coordinate of a point (this is a little harder to see, but a sketch will help you believe it). So, the images of our two points are $(0, -\frac{2}{3})$ and $(2,0)$, respectively.

The last step is to find the equation of the line through these two points. We already know the $y$-intercept; it's the point $(0, -\frac{2}{3})$. We can use the slope formula to find the slope.

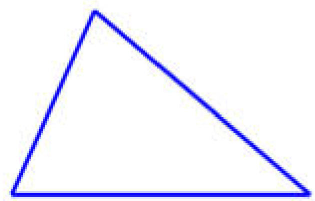
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0-(-\frac{2}{3})}{2-0} \\ &= \frac{\frac{2}{3}}{2} \\ &= \frac{1}{3} \end{aligned}$$

All told, the equation of the line after reflection is $y = \frac{1}{3}x - \frac{2}{3}$.

**5.** Describe all line and rotational symmetries (if any) of the following triangles. For line symmetries, draw a picture of the polygon together with all its lines of symmetry. For rotational symmetries, give the angle of the symmetry.

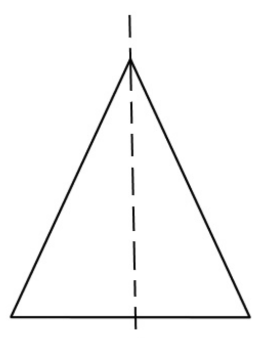
**5a.** scalene triangle

**Solution:** None of the sides of a scalene triangle are the same lenghth, so it has no line or rotational symmetries.



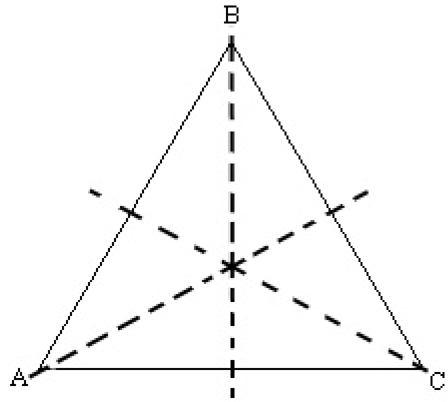
**5b.** isosceles triangle

**Solution:** An isosceles has two sides the same length, so they can be exchanged via line symmetry (see picture). There are no other symmetries.



**5c.** equilateral triangle

**Solution:** All the sides of an equilateral triangle are the same length, so we get a line symmetry for each vertex (see picture).




We also get two rotational symmetries. You can visualize them as rotating one vertex into another. If you pick up the shape and rotate vertex $A$ to where vertex $B$ currently is, that's one symmetry. You could also rotate vertex $A$ to where vertex $C$ currently is to get another. We don't count the trivial 360° rotation of sending vertex $A$ all the way back to itself.

To determine the angle of the symmetry (that is, the angle of the smallest turn that gives a rotational symmetry), we notice that it takes three turns to get $A$ back to $A$ ($A$ goes to $B$, then to $C$, then finally back to $A$). Since there are 360° in a full circle, each of these turns must be $\frac{360^\circ}{3} = 120^\circ$.

## Hints

**1a.** Slope of perpendicular line is the negative reciprocal of the given line. To find $b$, plug in a point and solve.

**1b.** Slope of perpendicular line is the negative reciprocal of the given line. To find $b$, plug in a point and solve.

**1c.** The line $x = 2$ is vertical, so a line perpendicular to it will be horizontal.

**1d.** The line $y = -6$ is horizontal, so a line perpendicular to it will be vertical.

**2a.** Just follow the recipe given in the translation.

**2b.** Just follow the recipe given in the translation.

**2c.** Look at the coordinates of $A$ and $A''$. How do the $x$-coordinates and $y$-coordinates change?

**2d.** Compare the translations you did in parts a and b to the translation you got in c.

**3.** If we let $(x, y)$ be the image of $(0, 4)$ after rotation, then we can make a right triangle whose vertices are $(0, 1)$, $(0, y)$, and $(x, y)$ (see the notes for an example). You know the hypotenuse and one of the angles, so you can use trigonometry to find $(x, y)$.

**4a.** Pick any two points on the line $y = 3x + 2$, reflect them both over the $x$-axis, then find the equation of the line passing through these two new points.

**4b.** Pick any two points on the line $y = 3x + 2$, reflect them both over the $y$-axis, then find the equation of the line passing through these two new points.

**4c.** Pick any two points on the line $y = 3x + 2$, reflect them both over the line $y = x$, then find the equation of the line passing through these two new points.

**5.** For all of these, draw a typical picture and think about which flips and turns will leave the shape unchanged. Be sure you don't impose more structure than you actually have. For example, if you are drawing an isosceles triangle, make sure you aren't drawing an equilateral triangle by mistake.