University of South Carolina
Math 222: Math for Elementary Educators II
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Section 002
Fall 2010

Quiz 3 (Version 2)

Due Wednesday, December 8

1. Suppose you drive a car at 60 miles per hour and never have to stop for gas.

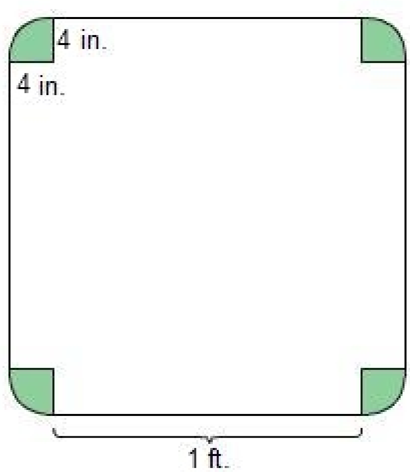
1a. How many miles do you travel in a year?

1b. Convert the distance in part a to kilometers.

1c. How long will it take a rocket traveling 60,000 km/hr to travel the distance in part b? Convert your answer to the most reasonable unit of time.

1d. This question was omitted from grading, so you don't have to correct it.

2. The figure below is a square with its corners rounded into perfect quarter-circles. Give your answers to the following in inches, square inches, or cubic inches, as appropriate.



2a. Find the perimeter of the shape.

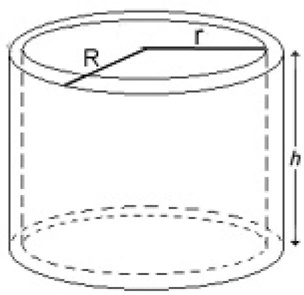
2b. Find the area of the shape.

2c. Find the volume of a 8 inch tall "cylinder" having this shape as its bases.

2d. Find the volume of a 8 inch tall "cone" having this shape as its base.

2e. Find the surface area of the "cylinder" in part c.

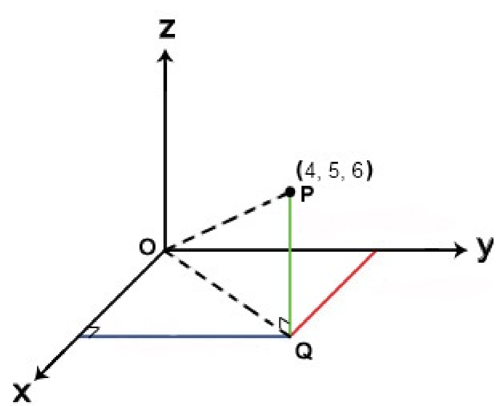
3. Suppose you have a cylinder of height h and radius R and you bore out a centrally-located cylinder of radius r. The result is called a cylindrical shell.



3a. Write an equation for the volume of the cylindrical shell in the picture.

3b. Write an equation for the surface area of the cylindrical shell in the picture.

4. Repeated application of the Pythagorean Theorem can be used to find the distance between points in higher dimensions.



4a. What is the length of the line segment $\overline{OQ}$?

4b. What is the length of the line segment $\overline{OP}$? (This is the distance between the origin and the point P.)

4c. This question was omitted from grading, so you don't have to correct it.

5. A circle has the points (-2, 3) and (5, -4) as opposite enpoints of one of its diameters.

5a. What is the radius of the circle?

5b. What is the center of the circle?

5c. What is the equation of the circle?

5d. Use your equation in part c to determine if the point (1, -1) lies on the circle.

Extra Credit

A. Mimicking the procedure in question 4, find an equation for the distance between *any* two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in three-dimensional space.

Hints

1a. Convert 60 miles per hour to miles per year.

1b. Use the fact that 1 kilometer equals .62 miles.

1c. Use distance = rate · time to find the time required. Convert this answer to an appropriate unit of time.

2a. Break the shape down into four lines and four quarter-circles. Find each length individually.

2b. Break the shape down into four rectangles, four quarter-circles, and a square. Find each area individually.

2c. The volume of a prism is the area of the base times the height of the prism.

2d. The volume of a cone is one-third of the volume of the related prism.

2e. Break the prism into the two bases (the shape in the picture) and the lateral face (which you can view as a rectangle if you make the appropriate cut).

3a. The volume of the shell is the volume of the big cylinder minus the volume of the small cylinder (the part in the middle that was removed).

3b. Break the shell into the two bases (which is a big circle minus a little circle), the outside lateral face (which can be viewed as a rectangle), and the inside "lateral face" (that is, the lateral face of the cylinder that was removed, which can also be viewed as a rectangle).

4a. Pythagorean Theorem

4b. Pythagorean Theorem

5a. The distance formula can tell you the distance between (-2, 3) and (5, -4). How can you get the radius from this distance?

5b. If you have a line segment with endpoints (x_1, y_1) and (x_2, y_2), then the midpoint is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ (this is often called the midpoint formula). Notice that what you're really doing is just averaging the x-coordinates and averaging the y-coordinates of the endpoints. This observation makes it easy to figure out the midpoint of a line segment in any number of dimensions (not just 2).

5c. Use the standard equation of the circle we developed in class and plug in your results from parts a and b.

5d. Does the given point satisfy your equation in part c?

A. Take the picture in problem 4 and replace the origin with the point (x_1, y_1, z_1) and the point $(2, 5, 3)$ with the point (x_2, y_2, z_2).