University of South Carolina
Math 222: Math for Elementary Educators II
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Section 002
Fall 2010

Quiz 3

Due Thursday, November 18

1. Astronomers use light-years to measure distances between celestial bodies. A light-year is the distance light travels in 1 year. The speed of light is (roughly) 300,000 km/sec.

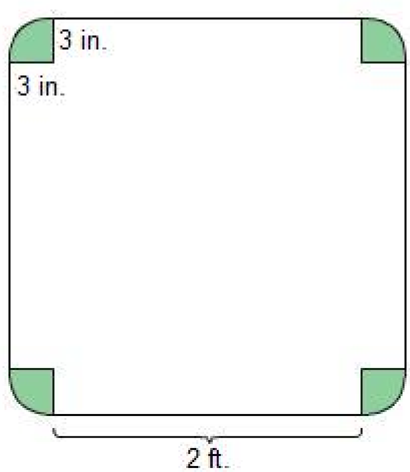
1a. How long is 1 light-year in kilometers?

1b. The nearest star (other than the sun) is Alpha Centauri. It is 4.34 light-years from Earth. How far is that in kilometers?

1c. How many years will it take a rocket traveling 60,000 km/hr to reach Alpha Centauri?

1d. How many days will it take the rocket in part c to travel to the sun if it takes approximately 8 minutes and 19 seconds for light from the sun to reach the earth?

2. The figure below is a square with its corners rounded into perfect quarter-circles. Give your answers to the following in inches, square inches, or cubic inches, as appropriate.



2a. Find the perimeter of the shape.

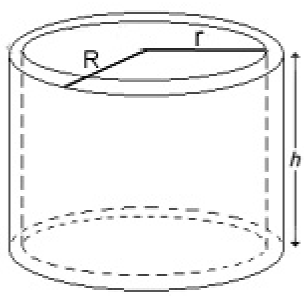
2b. Find the area of the shape.

2c. Find the volume of a 6 inch tall "cylinder" having this shape as its bases.

2d. Find the volume of a 6 inch tall "cone" having this shape as its base.

2e. Find the surface area of the "cylinder" in part c.

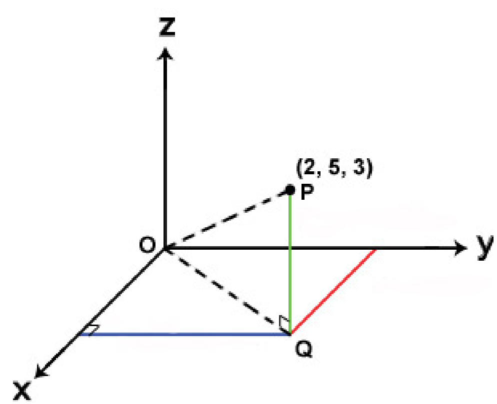
3. Suppose you have a cylinder of height h and radius R and you bore out a centrally-located cylinder of radius r. The result is called a cylindrical shell.



3a. Write an equation for the volume of the cylindrical shell in the picture.

3b. Write an equation for the surface area of the cylindrical shell in the picture.

4. Repeated application of the Pythagorean Theorem can be used to find the distance between points in higher dimensions.



4a. What is the length of the line segment $\overline{OQ}$?

4b. What is the length of the line segment $\overline{OP}$? (This is the distance between the origin and the point P.)

4c. Use this idea to find the distance between the origin and the point (2,5,3,7) in four-dimensional space.

5. A circle has the points (-1, 2) and (4, -3) as opposite enpoints of one of its diameters.

5a. What is the radius of the circle?

5b. What is the center of the circle?

5c. What is the equation of the circle?

5d. Use your equation in part c to determine if the point (1, -1) lies on the circle.

Extra Credit

A. Mimicking the procedure in question 4, find an equation for the distance between *any* two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in three-dimensional space.

<u>Hints</u>

1a. First, find the number of seconds in a year.

1b. Use your answer from part a.

1c. In part b, you found the distance to Alpha Centauri in kilometers. The ship covers 60,000 kilometers every hour, so you can use this to find the number of hours required to reach Alpha Centauri. Finally, convert this answer to years.

1d. Convert the time to seconds, then use the speed of light to find the distance between the sun and the earth. Finish the problem like in part c.

2a. Break the shape down into four lines and four quarter-circles. Find each length individually.

2b. Break the shape down into four rectangles, four quarter-circles, and a square. Find each area individually.

2c. The volume of a prism is the area of the base times the height of the prism.

2d. The volume of a cone is one-third of the volume of the related prism.

2e. Break the prism into the two bases (the shape in the picture) and the lateral face (which you can view as a rectangle if you make the appropriate cut).

3a. The volume of the shell is the volume of the big cylinder minus the volume of the small cylinder (the part in the middle that was removed).

3b. Break the shell into the two bases (which is a big circle minus a little circle), the outside lateral face (which can be viewed as a rectangle), and the inside "lateral face" (that is, the lateral face of the cylinder that was removed, which can also be viewed as a rectangle).

4a. Pythagorean Theorem

4b. Pythagorean Theorem

4c. In part a, you used the x-coordinate and the y-coordinate as the legs of a right triangle and found the hypotenuse. In part b, you used the answer from part a as one leg and the z-coordinate as the other leg, then found the associated hypotenuse. To extend to the fourth dimension, you can use the answer from part b as one leg and the fourth coordinate as the other leg, then find the associated hypotenuse.

5a. The distance formula can tell you the distance between (-1, 2) and (4, -3). How can you get the radius from this distance?

5b. If you have a line segment with endpoints (x_1, y_1) and (x_2, y_2), then the midpoint is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ (this is often called the midpoint formula). Notice that what you're really doing is just averaging the x-coordinates and averaging the y-coordinates of the endpoints. This observation makes it easy to figure out the midpoint of a line segment in any number of dimensions (not just 2).

5c. Use the standard equation of the circle we developed in class and plug in your results from parts a and b.

5d. Does the given point satisfy your equation in part c?

A. Take the picture in problem 4 and replace the origin with the point (x_1, y_1, z_1) and the point $(2, 5, 3)$ with the point (x_2, y_2, z_2).