

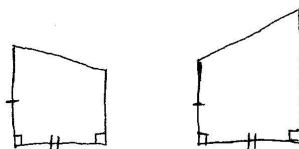
University of South Carolina
Math 222: Math for Elementary Educators II
Instructor: Austin Mohr
Section 002
Fall 2010

Quiz 2 Solutions

2. Determine which, if any, of the following congruence rules are true for quadrilaterals. If they are true, argue why you think they are true. If they are false, draw a pair of quadrilaterals (using a protractor and ruler) satisfying the rule but failing to be congruent.

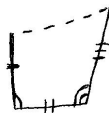
a. SASA

Solution: These are two quadrilaterals satisfying SASA, but they are not congruent.



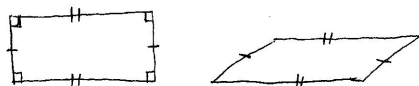
b. SASAS

Solution: Any quadrilateral satisfying SASAS will look like the picture below. The only way to complete the shape into a quadrilateral is to include the final edge (indicated by a dotted line), but this allows us no freedom. So, any two quadrilaterals satisfying SASAS will be congruent.



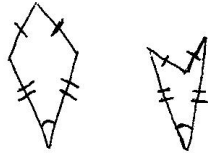
c. SSSS

Solution: These are two quadrilaterals satisfying SSSS, but they are not congruent.



d. SASSS

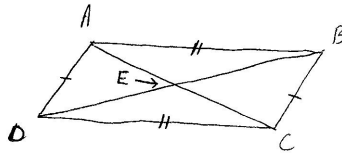
Solution: These are two quadrilaterals satisfying SASSS, but they are not congruent.



3. A *diagonal* of a quadrilateral is a line segment connecting opposite vertices (so every quadrilateral has exactly two diagonals).

a. Prove that the diagonals of a parallelogram bisect each other (that is, they cut each other perfectly in half).

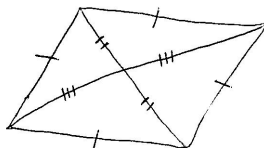
Solution: Drawing a parallelogram with its diagonals gives us this picture.



From the picture, you might suspect that each pair of opposite triangles are congruent. Our instinct is actually true; we just need to prove it. Let's try to show that triangle ABE is congruent to triangle DCE . We know that \overline{AB} is congruent to \overline{CD} since opposite sides in a parallelogram are congruent. We also know that angle $\angle AEB$ is congruent to angle $\angle CED$, since vertical angles are always congruent. Since \overleftrightarrow{AB} is parallel to \overleftrightarrow{CD} , we know that the opposite interior angles $\angle ACD$ and $\angle BAE$ are congruent. Taking all this together, we've established AAS congruence, so triangle ABE is congruent to triangle DCE . In particular, that means $AE = CE$ and $DE = BE$, which is what we wanted to prove.

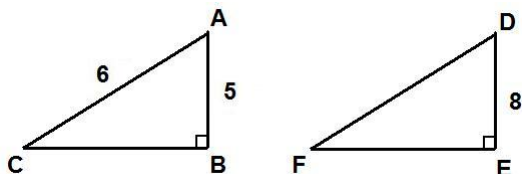
b. Prove that the diagonals of a rhombus are perpendicular (that is, they intersect at right angles).

Solution: Drawing a parallelogram with its diagonals gives us this picture.



Notice that, since every rhombus is a parallelogram, part a tells us that the diagonals bisect each other. From this picture, we already have SSS congruence for all four triangles. That is, all four triangles are congruent to one another. In particular, the four angles formed where the diagonals cross are all congruent. Since there is a total of 360 degrees formed by those four angles and each angle has the same measure, we know that each angle is $\frac{360}{4} = 90$ degrees. In other words, the diagonals are perpendicular.

4. Solve for all missing sides and angles given that the two triangles below are similar and oriented the same way (so \overline{AB} really is associated with \overline{DE}).



Solution: There are many ways to approach this problem. The easiest is probably to solve the triangle on the left completely, then transfer the information to the triangle on the right.

$$\begin{aligned}\cos(\angle A) &= \frac{5}{6} \\ \cos^{-1}(\cos(\angle A)) &= \cos^{-1}\left(\frac{5}{6}\right) \\ \angle A &= 33.6^\circ\end{aligned}$$

$$\begin{aligned}\angle C &= 180^\circ - 90^\circ - 33.6^\circ \\ &= 56.4^\circ\end{aligned}$$

$$\begin{aligned}\sin(33.6^\circ) &= \frac{BC}{6} \\ .553 &= \frac{BC}{6} \\ 3.32 &= BC\end{aligned}$$

In similar triangles, associated angles are all the same, so we know $\angle D = 33.6^\circ$ and $\angle F = 56.4^\circ$ without any work. To find the other sides, we could run through all the trig work again, or we could find the scale factor that turns a side in the triangle on the left into a triangle on the right. We can do this by comparing side \overline{DE} and \overline{AB} .

$$\begin{aligned}\frac{DE}{AB} &= \frac{8}{5} \\ &= 1.6\end{aligned}$$

So, sides in the triangle on the right are 1.6 times bigger than sides in the triangle on the left. This means

$$\begin{aligned}DF &= 1.6 \cdot AC \\ &= 1.6 \cdot 6 \\ &= 9.6\end{aligned}$$

and

$$\begin{aligned}EF &= 1.6 \cdot BC \\ &= 1.6 \cdot 3.32 \\ &= 5.31.\end{aligned}$$

5. Two different companies charge different amounts for service. Company A charges a \$200 activation fee and \$60 per month. Company B charges a \$100 activation fee and \$70 per month.

- a. Write two equations (one for each company) to describe the cost of using their services. Be sure to specify what your x and y mean.

Solution: You can tell this situation will use a linear equation because it involves repeated addition. (If you took Math 111, you might recall that there is another kind of formula for repeated multiplication. In practice, you

would need to decide which, if any, of these formulas is appropriate for the situation you are trying to model.) Every linear equation looks like $y = mx + b$. The value for m is the thing that's being added with every unit of time (x) that passes. The value for b is the initial value that gets added in once and never again. In this problem, m will be the monthly rate and b will be the activation fee. I will let the variable x be the number of months that have passed and y be the total cost for x months of service. So, the first company is modeled by

$$y = 60x + 200$$

and the second by

$$y = 70x + 100.$$

- b. Sketch the graph of your two equations together on the same set of axes.

Solution:

- c. Where do the lines intersect? What is the meaning of this point in context?

Solution: You can find where two lines intersect by setting their equations equal to each other and solving for x .

$$60x + 200 = 70x + 100$$

$$100 = 10x$$

$$10 = x$$

So, they intersect when $x = 10$, but we also need to know what y is at that time. We can just plug $x = 10$ into either equation to find out.

$$\begin{aligned} y &= 60x + 200 \\ &= 60(10) + 200 \\ &= 800 \end{aligned}$$

So, the lines intersect at $(10, 800)$. In this situation, that means that 10 months of service costs \$800 no matter which company you choose.

- d. Compare the prices for one year of service from each of the companies.

Solution: One year of service is 12 months, so we can just substitute $x = 12$

into each equation.

$$\begin{aligned}y &= 60x + 200 \\ &= 60(12) + 200 \\ &= 920\end{aligned}$$

$$\begin{aligned}y &= 70x + 100 \\ &= 70(12) + 100 \\ &= 940\end{aligned}$$

So, one year of service costs \$920 with the first company and \$940 with the second.