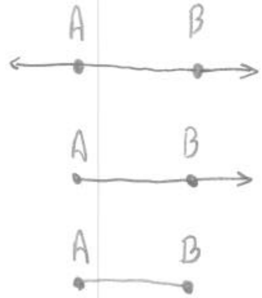
11.1 - Basic Definitions

point 

no dimensions

Given two points, we can form

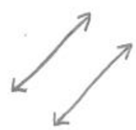


line $\overleftrightarrow{AB}$

ray $\overrightarrow{AB}$

line segment $\overline{AB}$

If two lines have no points in common, and belong to the same plane they are parallel.

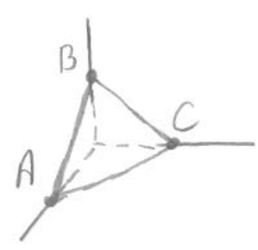


Otherwise, they intersect.

Given three points, we might get a line.

A, B, C collinear points

If they don't all lie all the same line, we can form a plane.



plane ABC

Imagine a plane as being a sheet of paper floating in space, but it has no boundary (just like a line has no endpoints).

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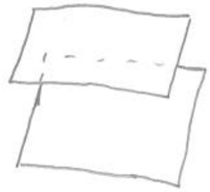
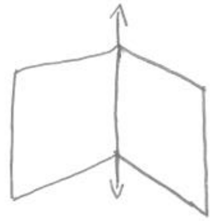
Just like lines, planes can be



<u>parallel</u> (like the way a floor and ceiling do not intersect)

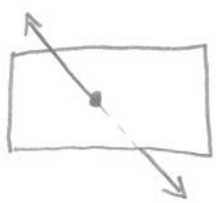


<u>intersecting</u> (like the way two walls intersect)

A line can be

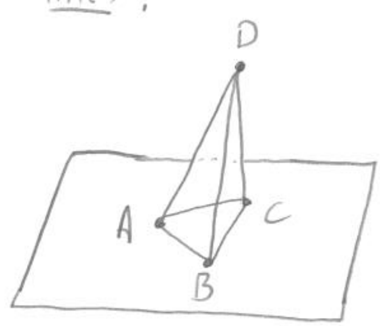
<u>parallel</u> to a plane

lie <u>inside</u> a plane



or <u>intersect</u> a plane at one point

If two lines cannot be contained in a single plane, they are called <u>skew lines</u>.

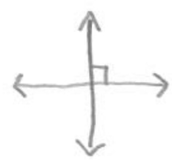


$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are skew

Angles are a measure of openness between two rays having a common endpoint.

straight angle — right angle — acute angle (less than 90°) — obtuse angle (more than 90°)

If two lines happen to intersect at right angles, they are called perpendicular lines.



Notice all the angles are 90° if even one of them is.

Planes can also be perpendicular (for example, a floor and a wall).

Ex

$\angle CAB$ is 30° (the angle formed by $\vec{AC}$ and $\vec{AB}$)

$\angle DAB$ is 90° (the angle formed by $\vec{AD}$ and $\vec{AB}$)

What is the measure of $\angle DAC$?

Call this missing measurement x. From the picture, we get

$$90° = 30° + x$$

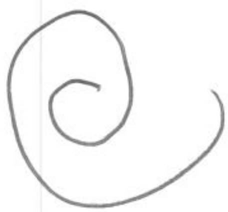
$$60° = x$$

So, $\angle DAC$ is 60°.

9.2 - Polygons

1. composed of line segments

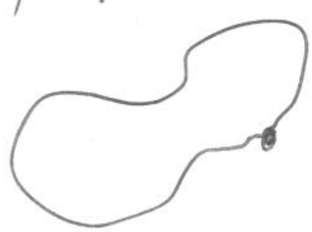
not composed of line segments

2.  

not closed

want shapes closed

starting point is same as ending point



this is closed

Is this enough?

want to avoid this

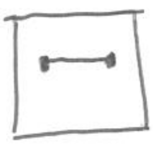
3. No intersecting lines (called simple)

simple

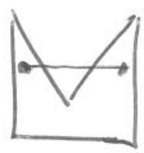
not simple

A polygon is simple, closed, and made of line segments.

What's different about these?



convex

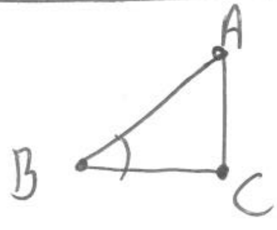


concave

Convex means if take any two points in interior, connect them. The line segment is in the interior. (Also simple and closed.)

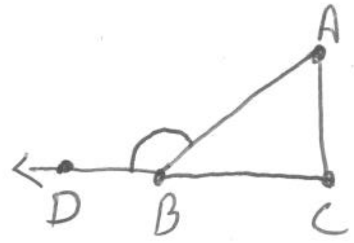
Concave means simple, closed, but not necessarily having the line property.

Interior angles



$\angle ABC$

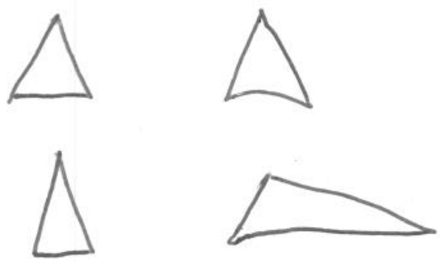
Exterior angle



$\angle ABD$

When are two polygons the same?

need same number of sides



even with same number of sides, these are very different

1.

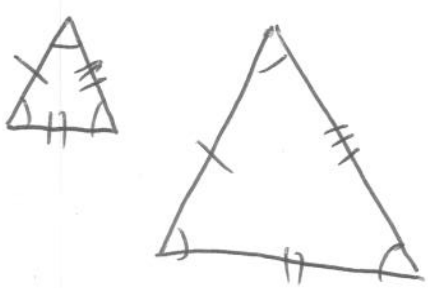
$\angle ABC = \angle DEF$

$\angle BCA = \angle EFD$

$\angle BAC = \angle EDF$

You're allowed to rotate and flip triangles so long as the angles don't change.

2.



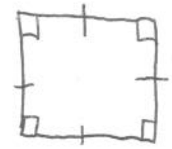
In the same way as angles, want pairs of line segments equal.

Say two polygons are <u>congruent</u> (the same) when their sides and angles match up like above.

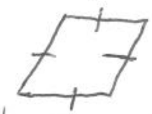
If all sides of a single polygon happen to be the same length, then it's called regular.

and all angles the same measure

regular triangle



regular quadrilateral (square)



rhombus not regular

Triangles

Right triangle - Has a 90° angle.

Acute triangle - All angles less than 90°

Obtuse triangle - Has an obtuse angle.

Isosceles triangle - Has at least two sides same length.

Equilateral triangle - All sides the same

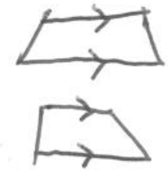
Scalene triangle - No two sides equal

Quadrilaterals (four sided)

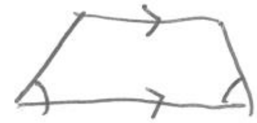
~~Square~~

Parallelogram - Each pair of opposite sides are parallel.

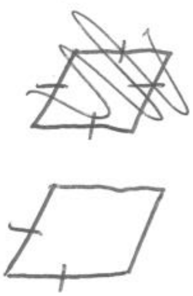
Trapezoid - At least one pair of parallel sides



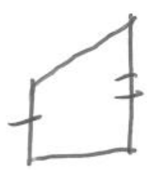
Isosceles Trapezoid - Trapezoid with ~~base~~ equal base angles



Rhombus - Parallelogram with two adjacent sides equal

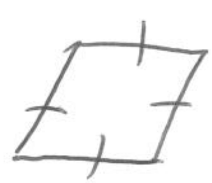


Can we say more about rhombus?



top and bottom not parallel

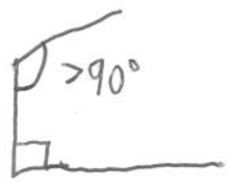
So, actually know



Kite - Two pairs of equal adjacent sides

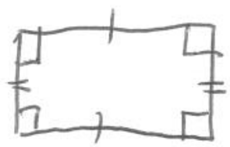
Rectangle – Parallelogram with one right angle.

Can we say more?



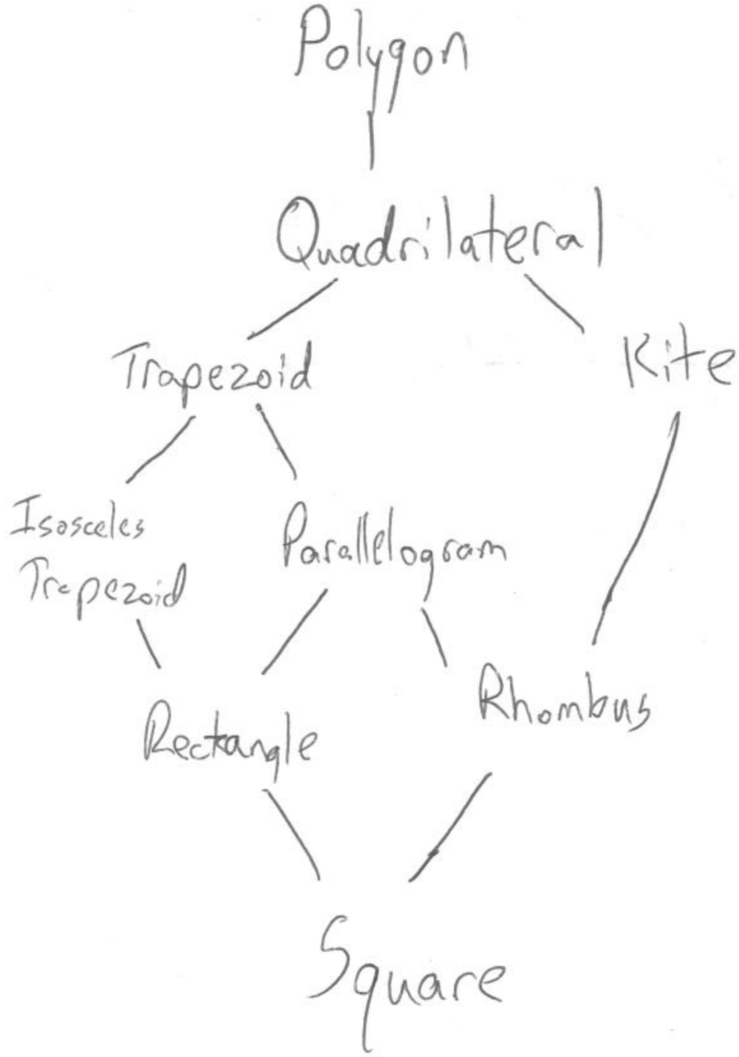
If not 90°, then sides aren't parallel.

Find out



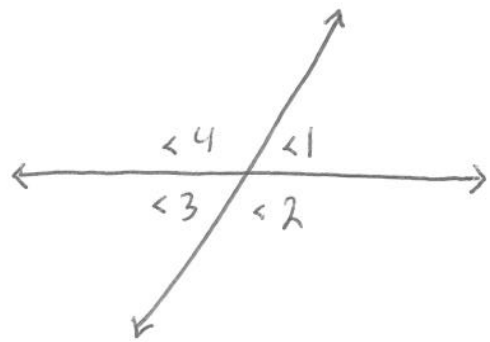
Square – Rectangle with two adjacent sides equal

Can deduce


Polygon
Quadrilateral
Trapezoid
Kite
Isosceles Trapezoid
Parallelogram
Rectangle
Rhombus
Square

9.3 - Angles

Given two intersecting lines and one angle measurement.



$m(\angle 1) = 60°$

We can find the other measurements.

- $180° = m(\angle 1) + m(\angle 2)$

  $180° = 60° + m(\angle 2)$

  $120° = m(\angle 2)$

- $180° = m(\angle 2) + m(\angle 3)$

  $180° = 120° + m(\angle 3)$

  $60° = m(\angle 3)$

- $180° = m(\angle 3) + m(\angle 4)$

  $180° = 60° + m(\angle 4)$

  $120° = m(\angle 4)$

Pairs of angles like <1 and <3 or <2 and <4 are called <u>vertical angles</u>.

In the previous example, we saw that the pairs of vertical angles had equal measure. This is always the case.

<u>Theorem</u> If two angles are vertical, then they have the same measure.

<u>Proof</u>

Suppose $m(<1) = x^\circ$. We want to show that $m(<3) = x^\circ$. We know

$$180^\circ = m(<1) + m(<2)$$

$$180^\circ = x^\circ + m(<2)$$

$$180^\circ - x^\circ = m(<2)$$

We also know

$$180^\circ = m(<2) + m(<3)$$

$$180^\circ = (180^\circ - x^\circ) + m(<3)$$

$$0^\circ = -x^\circ + m(<3)$$

$$x^\circ = m(<3)$$

$\square$

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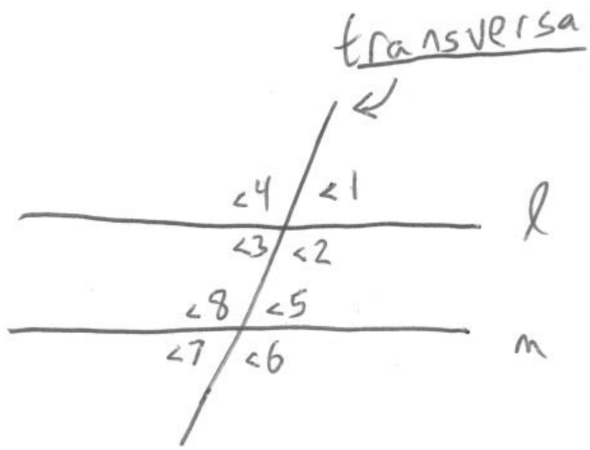
Some words

- Two angles are <u>supplementary</u> if their measures sum to $180°$.
- " <u>complementary</u> " sum to $90°$.

<u>Transervals</u>



l parallel to m

<u>Interior</u> <u>angles</u>: ∠2, ∠3, ∠5, ∠8

<u>Exterior</u> <u>angles</u>: ∠1, ∠4, ∠7, ∠6

<u>Alternate</u> <u>interior</u>: ∠2 and ∠8, ∠3 and ∠5

<u>Alternate</u> <u>exterior</u>: ∠1 and ∠7, ∠4 and ∠6

<u>Corresponding</u>: ∠1 and ∠5, ∠2 and ∠6, ∠4 and ∠8, ∠3 and ∠7

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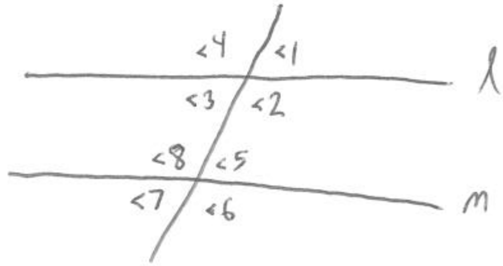
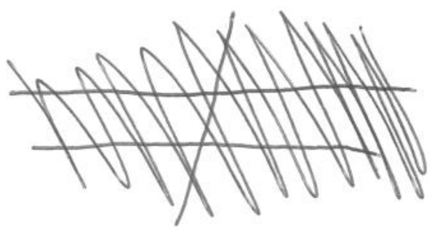


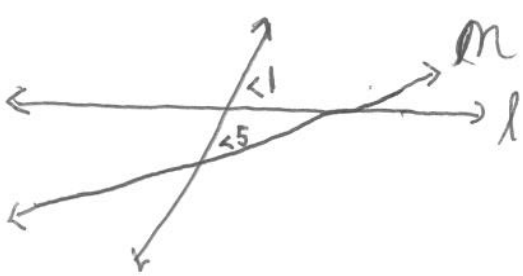
l parallel to m

Suppose $m(\angle 1) = 60°$.

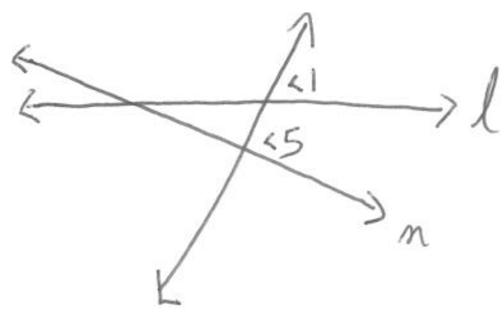
- $m(\angle 3) = 60°$, since $\angle 1$ and $\angle 3$ are vertical
- $m(\angle 2) = m(\angle 4) = 120°$, since $\angle 1$ and $\angle 2$ are supplementary (as are $\angle 1$ and $\angle 4$)

It looks like $m(\angle 5)$ should **be** the same as $m(\angle 1)$.





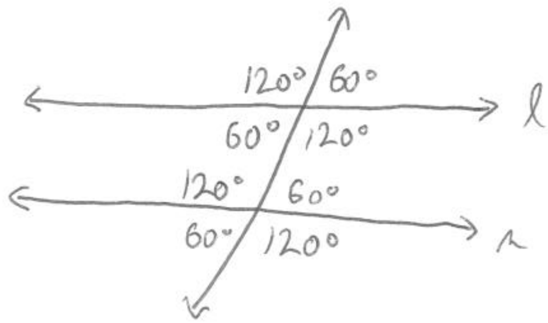
If $m(\angle 5)$ too small, l and m intersect.



If $m(\angle 5)$ too big, l and m intersect.

Since l and m are supposed to be parallel, $m(\angle 1)$ and $m(\angle 5)$ must be equal.

So, just knowing one angle, we get



l is parallel to m

We've just demostrated that all pairs of

vertical angles
alternate interior angles
alternate exterior angles
corresponding angles

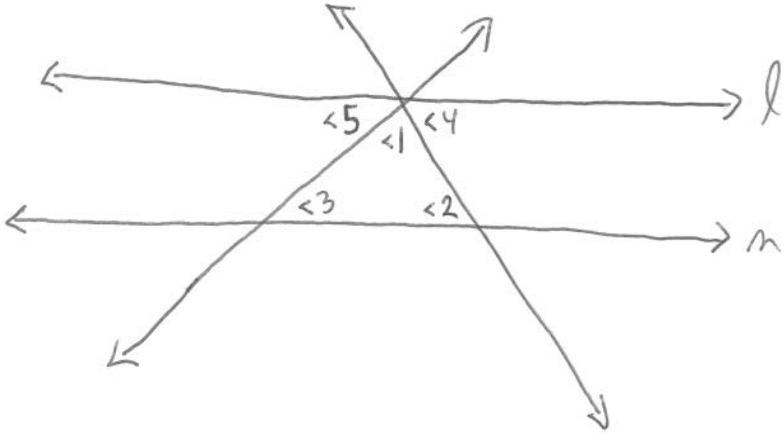
have equal measure.

<u>Theorem</u> The sum of the measures of the interior angles of a triangle is 180°.

<u>Proof</u>

We want $m(\angle 1) + m(\angle 2) + m(\angle 3) = 180°$.

If we turn the picture into one about transversals, we can learn more.



l parallel to m

A few things to notice

- $m(\angle 1) + m(\angle 4) + m(\angle 5) = 180°$ (they are supplementary)
- $m(\angle 2) = m(\angle 4)$ (they are alternate interior)
- $m(\angle 3) = m(\angle 5)$ (they are alternate interior)

Put it all together

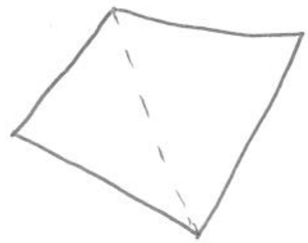
$$180° = m(\angle 1) + m(\angle 4) + m(\angle 5)$$

$$180° = m(\angle 1) + m(\angle 2) + m(\angle 5) \qquad (m(\angle 2) = m(\angle 4))$$

$$180° = m(\angle 1) + m(\angle 2) + m(\angle 3) \qquad (m(\angle 3) = m(\angle 5))$$

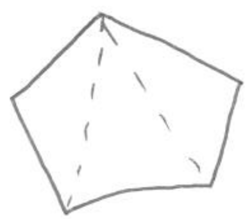
$\square$

What about other polygons?



Quadrilateral

Sum of interior angles of quadrilateral
= Sum of interior angles of two triangles
= $2 \cdot 180°$



Pentagon

Sum of interior angles of pentagon
= Sum of interior angles of three triangles
= $3 \cdot 180°$

<u>Theorem</u> The sum of the measures of the interior angles of an n-gon (a polygon with n sides) is $(n-2) \cdot 180°$.

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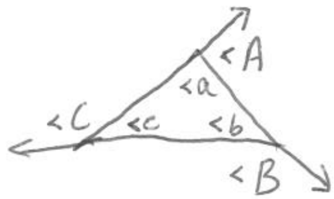
What about exterior angles?



$$(m(\angle a) + m(\angle A)) + (m(\angle b) + m(\angle B)) + (m(\angle c) + m(\angle C))$$

$$= 180° + 180° + 180°$$

$$= 3 \cdot 180°$$

$$m(\angle a) + m(\angle b) + m(\angle c) = 1 \cdot 180°$$

$$\cancel{m(\angle a)}$$

$$(m(\angle a) + m(\angle A)) + (m(\angle b) + m(\angle B)) + (m(\angle c) + m(\angle C))$$
$$-(m(\angle a) + m(\angle b) + m(\angle c))$$

$$= 3 \cdot 180° - 1 \cdot 180°$$

$$= 2 \cdot 180°$$

$$= 360°$$

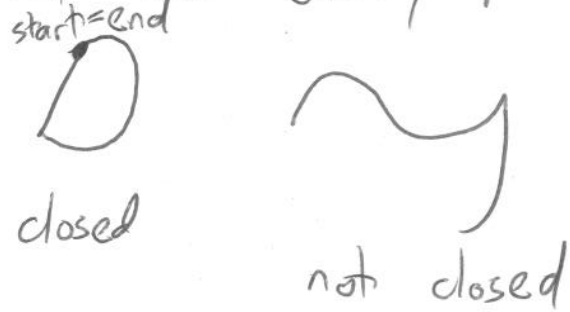
This remains true for any polygon.

<u>Theorem</u> The sum of the measures of the exterior angles of any polygon is always 360°.

9.4 - Polyhedra

Simple curve - no intersection

Closed curve - start point and ending point same

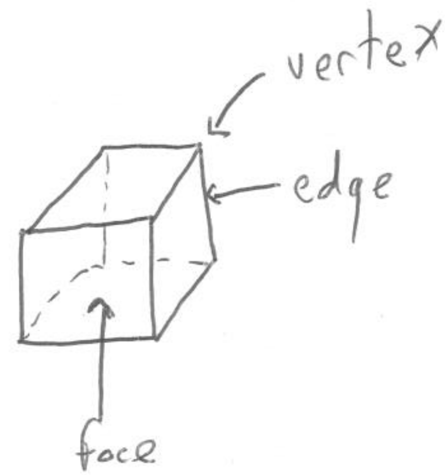


What are these in 3d?

Simple surface - exactly one interior

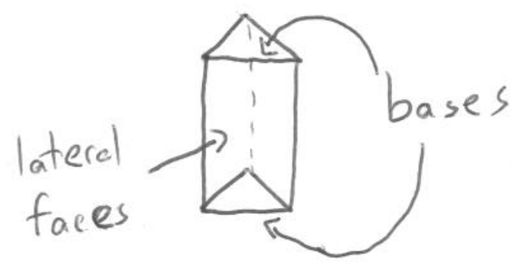
Closed surface - no holes in surface

Polyhedron is simple, closed surface with polygon faces.



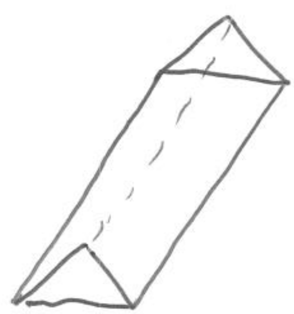
Kinds of polyhedra

- **Prism** - Two congruent, parallel polygons, connect corresponding vertices.



right triangular prism

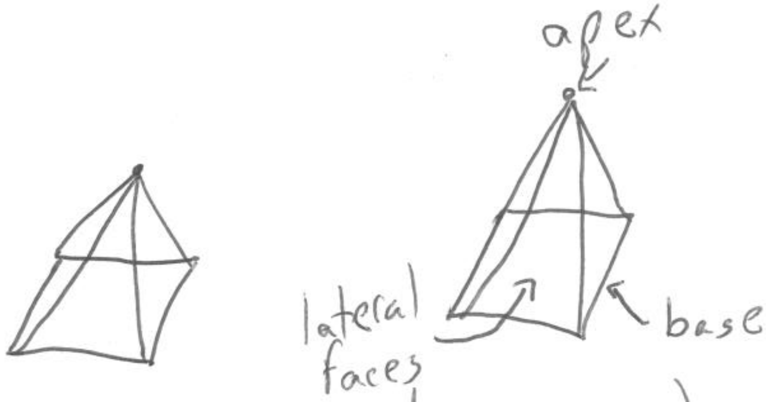
means lateral faces are rectangles



oblique triangular prism

means lateral faces parallelograms but not rectangles

- Pyramid



Polygon base and a single point (apex) above the base and connect it to all vertices of base

- Regular ~~polyd~~ polyhedra

Regular polygon –

all sides equal (actually, all angles equal, too)

Regular polyhedron

- All faces are regular polygons and congruent to each other
- The number of edges meeting at any vertex is equal everywhere.

How many regular polyhedra?

angles start out small

angles get wider

Eventually, the edges are flat on floor.

So, when edges are flat on the floor, the angles sum to 360°.

At each vertex of a regular polyhedron, ~~there~~ the angles sum to less than 360°.

Face	Faces at each Vertex	Sum of Angles at each vertex	
☆ Triangle	3	180°	okay

Equilateral triangle

180° total, so 60° at each angle

~~Triangle~~

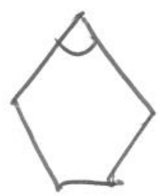
Face	Faces at each vertex	Sum of angles at each vertex	
☆ Triangle	4	240°	okay
☆ Triangle	5	300°	okay
Triangle	6	360°	no good
☆ Square	3	270°	okay
Square	4	360°	no good
☆ Pentagon	3	324°	okay
Pentagon	4	more than 360°	no good
Hexagon	3	360°	no good

Pentagon

Total degrees of interior angles

$$3 \cdot 180° = 540°$$

(In general, an n-gon has $(n-2) \cdot 180°$)



Each angle is $\frac{540°}{5} = 108°$

Hexagon

$$(6-2) \cdot 180° = 720°$$

So each angle is $\frac{720°}{6} = 120°$