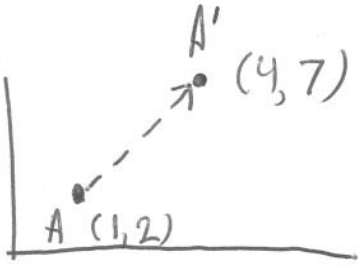
12.1 - Translation and Rotation

Translate A to A'.



Move right 3 $(4-1=3)$

up 5 $(7-2=5)$

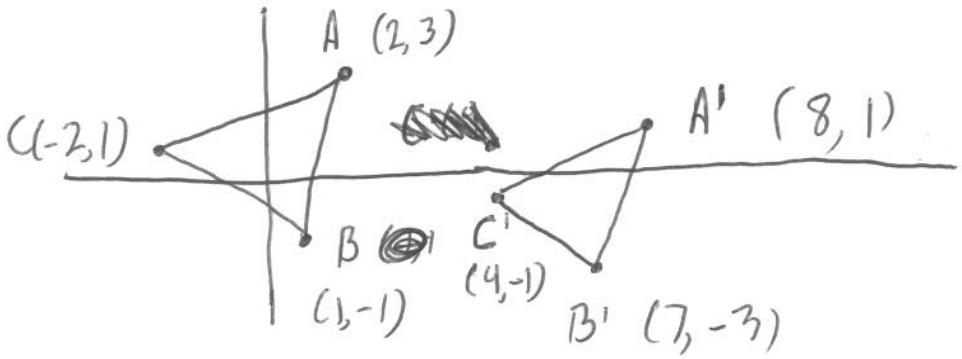
or

$(x, y) \rightarrow (x+3, y+5)$

or

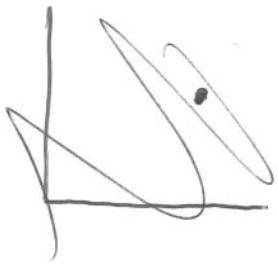
$A(1,2)$ to $A'(4,7)$

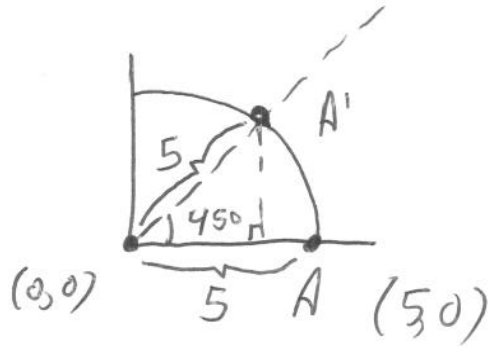
Translate by $(x,y) \rightarrow (x+6, y-2)$.

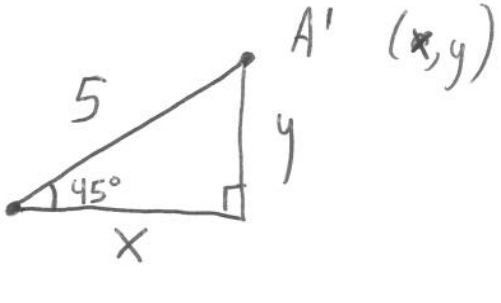


Rotation

Rotate A 45° counterclockwise around the origin.







So A' is at (3.55, 3.55)

Find x

$\cos(45°) = \frac{x}{5}$

$.71 = \frac{x}{5}$

$3.55 = x$

Find y

$\sin(45°) = \frac{y}{5}$

$.71 = \frac{y}{5}$

$3.55 = y$

or

$x^2 + y^2 = 5^2$

$3.55^2 + y^2 = 5^2$

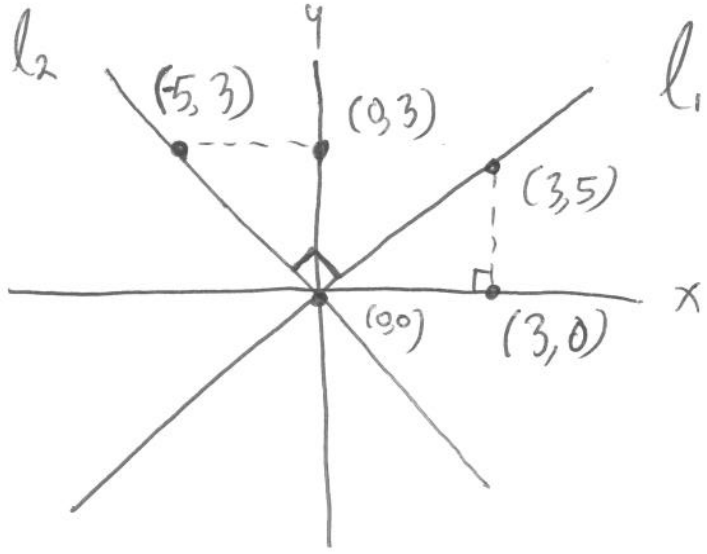
$12.6 + y^2 = 25$

$y^2 = 12.4$

$\sqrt{y^2} = \sqrt{12.4}$

$y = 3.52$

Perpendicular Lines



l_1 and l_2 are perp means they intersect at a right angle.

Another way to say it is that l_1 has been rotated 90° counterclockwise.

What is slope of l_1?

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5-0}{3-0}$$

$$= \frac{5}{3}$$

What is slope of l_2?

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5-0}{3-0}$$

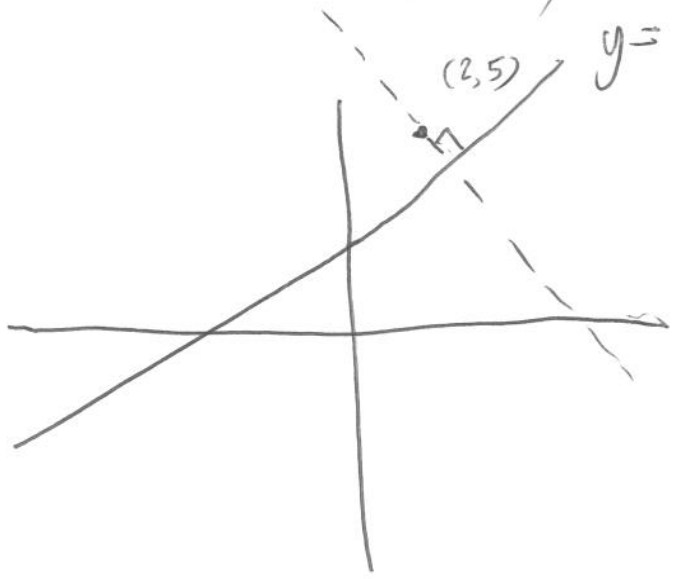
$$= \frac{-5}{3}$$

$$= \frac{3-0}{-5-0}$$

$$= -\frac{3}{5}$$

☆ Perpendicular lines have negative reciprocal slopes.

Ex Find equation of line perpendicular to $y = \frac{2}{3}x + 3$ and passing through $(2, 5)$.



$y = mx + b$

$m = \frac{-3}{2}$

$y = \frac{-3}{2}x + b$

Plug in $(2, 5)$

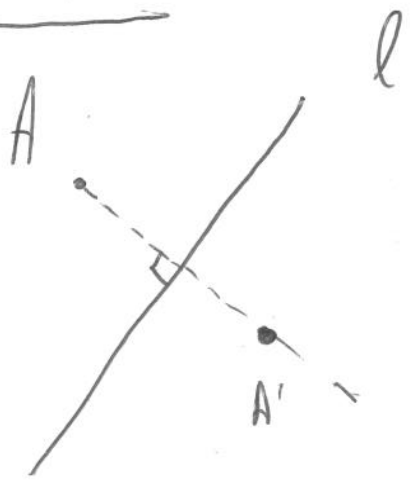
$5 = \frac{-3}{2} \cdot 2 + b$

$5 = -3 + b$

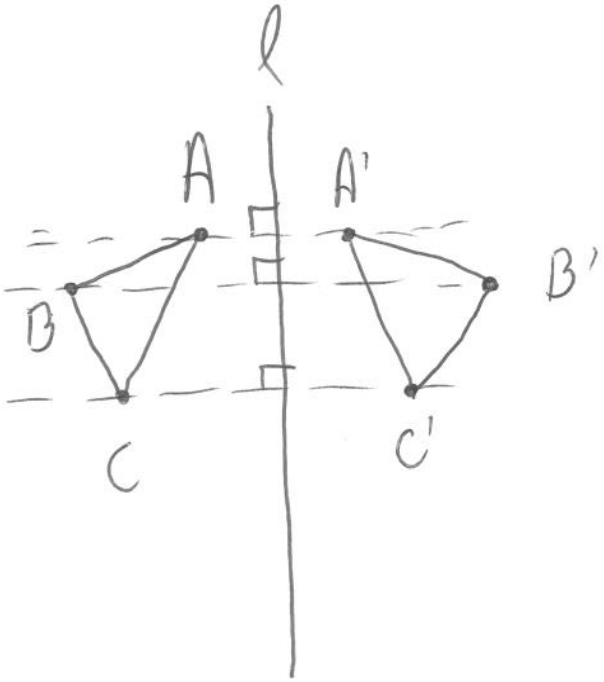
$8 = b$

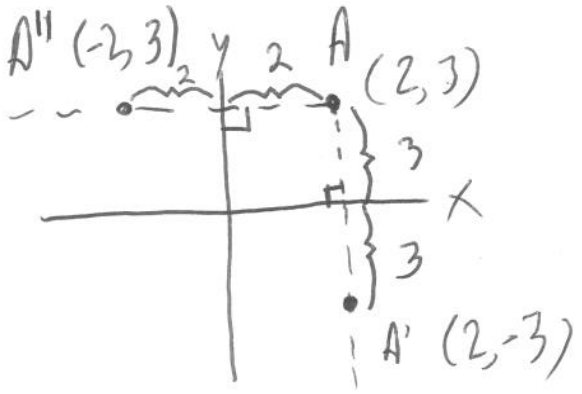
$$y = \frac{-3}{2}x + 8$$

Reflections

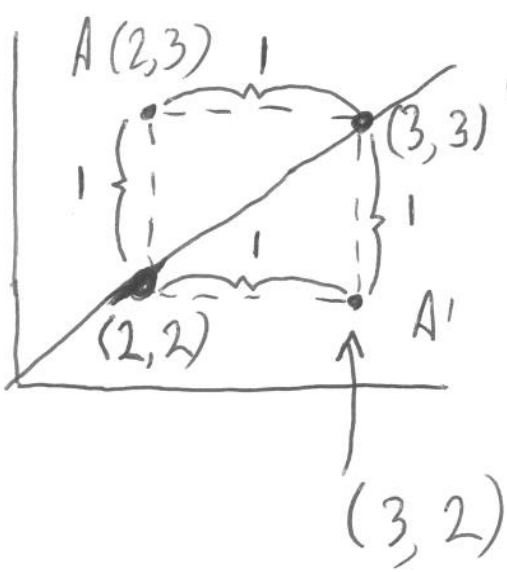


Reflect A over ℓ.
Draw a line through A perpendicular to ℓ.
Distance from A to ℓ is same as distance from A' to ℓ.

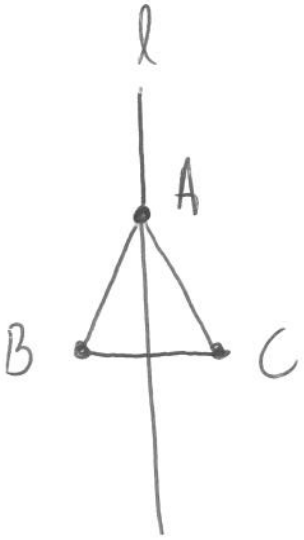




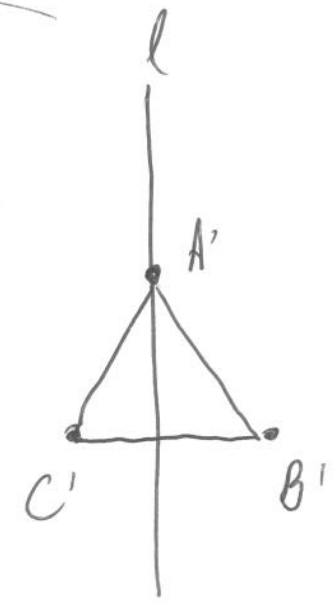
Reflect A over x-axis and y-axis.



Reflect A over $y = x$.



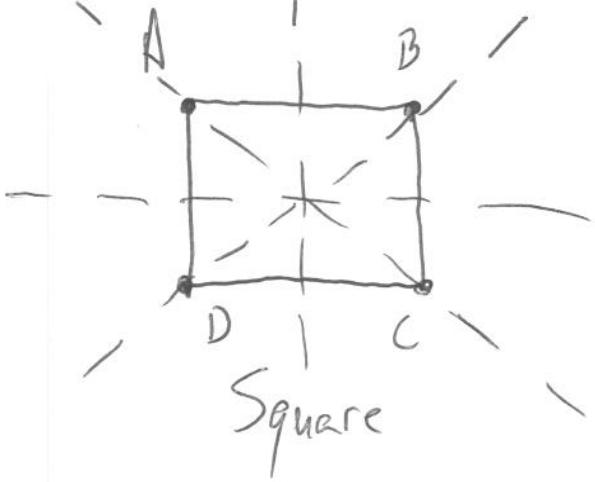
Reflect →



Equilateral Triangle

Get the same shape

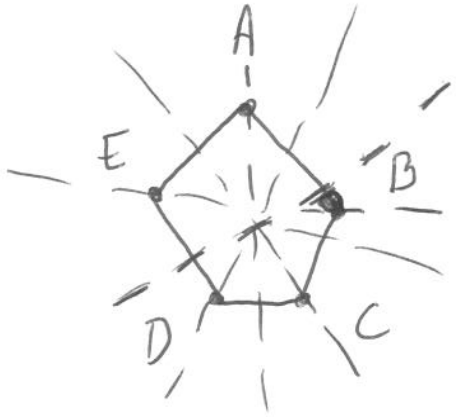
Say the triangle is symmetric about line l.



Square

How many line symmetries?

4

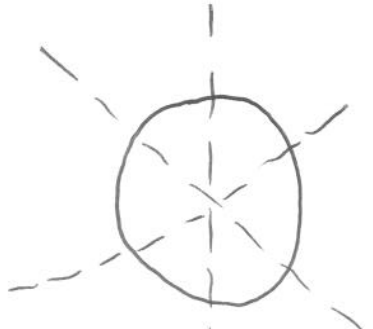


Regular Pentagon

How many line symm.?

5

Regular n-gon has n line symmetries.



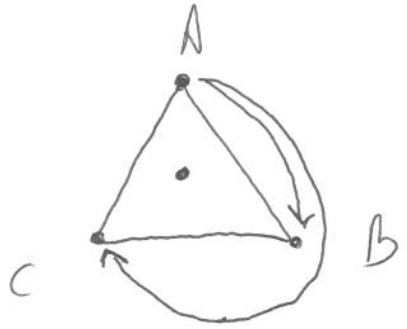
Circle

Infinite line symmetries.

↑

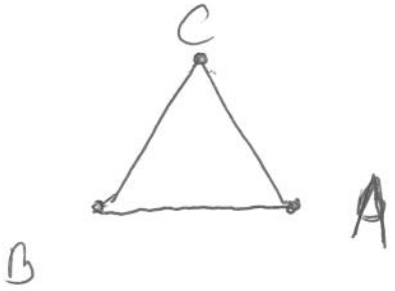
Unbounded number

Rotational Symmetry

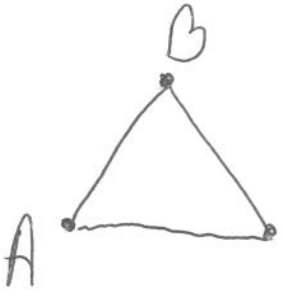


Equilateral Triangle

Rotate →



120° turn



240° turn

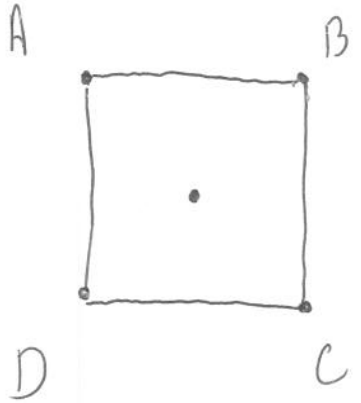
Two rotational symmetries.

What is the angle of the symmetry?

3 rotations sends A back to itself.

360° in a circle.

So, $\frac{360^\circ}{3} = 120^\circ$ in each turn.

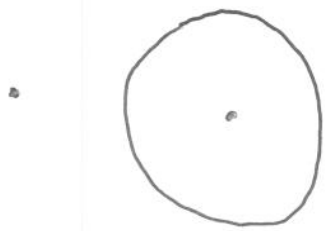


How many rotational symmetries?

3

What is the angle?

$$\frac{360^\circ}{4} = 90^\circ$$



circle

Infinite rotational symmetries