University of South Carolina
Math 221: Math for Elementary Educators
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Section 001
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<u>Test 3</u>

1. Adding Decimals

a. Write 3.8 and 4.26 in expanded form.

<u>Solution</u>: The first number is "3 and 8 tenths", so the expanded form is $3 + \frac{8}{10}$. The second number is "4, 2 tenths, and 6 hundredths", so the expanded form is $4 + \frac{2}{10} + \frac{6}{100}$.

b. Use the expanded forms in part a to find $3.8 + 4.26$. Write your final answer as a decimal.

<u>Solution</u>: Since we can add in any order we like, we get

$$\begin{aligned} 3.8 + 4.26 &= \left(3 + \frac{8}{10}\right) + \left(4 + \frac{2}{10} + \frac{6}{100}\right) \\ &= (3+4) + \left(\frac{8}{10} + \frac{2}{10}\right) + \frac{6}{100} \\ &= 7 + \frac{10}{10} + \frac{6}{100} \\ &= (7+1) + \frac{6}{100} \\ &= 8 + \frac{6}{100} \\ &= 8.06. \end{aligned}$$

c. Why must you line up the decimal point to use the standard algorithm?

<u>Solution</u>: Lining up the decimal point when using the standard algorithm makes sure that we add like place values (units to units, tenths to tenths, hundredths to hundredths, etc.).

2. Multiplying Decimals

a. Convert 3.2 and 0.21 to fractions (*not* mixed numbers). Do not reduce.

Solution: To get from 32 to 3.2, we need to divide by 10. So, $3.2 = \frac{32}{10}$. To get from 21 to 0.21, we need to divide by 100. So, $0.21 = \frac{21}{100}$.

b. Use the fractions in part a to find $3.2 \cdot 0.21$. Write your final answer as a decimal.

Solution: Using the fraction forms from part a, we get

$$\begin{aligned} 3.2 \cdot 0.21 &= \frac{32}{10} \cdot \frac{21}{100} \\ &= \frac{32 \cdot 21}{10 \cdot 100} \\ &= \frac{672}{1000} \\ &= 0.672. \end{aligned}$$

c. According to the standard algorithm, how many digits should be behind the decimal in your answer? Why must this be the case? (Simply restating the rule will not get full credit. I want to know *why* the rule is the way it is.)

Solution: In part b, there is one digit behind the decimal in 3.2 and two digits behind the decimal in 0.21. The rule is that the product should have $1 + 2 = 3$ digits behind the decimal place. This is because, in the fraction form of 3.2, we have a denominator of 10 and, in the fraction form of 0.21, we have a denominator of 100. When we do the multiplication, we get a denominator of 1000. Dividing a number by 1000 moves the decimal point three places to the left, which gives us three digits behind the decimal.

3. Percents

a. You baked 240 cookies for a bake sale and sold $\frac{3}{4}$ of them. How many cookies did you sell?

Solution:

$$\begin{aligned} \text{percent} &= \frac{\text{part}}{\text{whole}} \\ \frac{3}{4} &= \frac{\text{part}}{240} \\ \frac{3}{4} \cdot 240 &= \text{part} \\ 180 &= \text{part} \end{aligned}$$

So, 180 cookies were sold.

b. You contribute \$18 toward the cost of dinner, which happens to be 15% of the total bill. What is the total cost of the dinner?

Solution:

$$\begin{aligned} \text{percent} &= \frac{\text{part}}{\text{whole}} \\ 0.15 &= \frac{18}{\text{whole}} \\ 0.15 \cdot \text{whole} &= 18 \\ \text{whole} &= 18 \div 0.15 \\ \text{whole} &= 120 \end{aligned}$$

So, the total cost of the dinner was \$120.

4. You survey some of your friends about terrible music and find that:

- 12 hate Nickelback
- 11 hate John Mayer
- 5 hate both Nickelback and John Mayer

Let N and J be the set of surveyed people that hate Nickelback and John Mayer, respectively.

a. What do people in the set $N \cap J$ think about Nickelback and John Mayer?

Solution: People in $N \cap J$ belong to N and J at the same time, so these people hate both Nickelback and John Mayer. Fun Fact: Your instructor belongs to $N \cap J$.

b. What do people in the set $N - J$ think about Nickelback and John Mayer?

Solution: People in $N - J$ belong to N but *not* J, so these people hate Nickelback, but do not hate John Mayer.

c. Draw a Venn Diagram to represent the results of the survey.

Solution: We start by putting 5 people in the middle ($N \cap J$), since we know there's know double-counting weirdness going on there. Now, what about the leftmost portion ($N - J$)? We know that a total of 12 people hate Nickelback, but 5 of them have already been accounted for. So, there are 7 people in $N - J$. What about the rightmost part ($J - N$)? We know that a total of 11 people hate John Mayer, but 5 of them have already been accounted for. So, there are 6 people in $J - N$.

5. Each of the following statements is false. For each one, state the negation and prove that it is true.

a. Some natural numbers are negative.

Solution: The negation is "Every natural number is not negative.". You might have also come up with more natural-sounding (but equivalent) statements like "Every natural number is non-negative." or "No natural number is negative.". In any case, the negation is a true statement; the natural numbers are the numbers $1, 2, 3, \ldots$, none of which are negative.

b. If 4 is even, then 5 is even.

Solution: The negation is "4 is even and 5 is not even.". If you like to use flowery language in your math (like me), then you might say "4 is indeed even, yet 5 is not even.". The negation is obviously true; both "4 is even" and "5 is not even" are true statements.

6. The following statement is true.

If $x \cdot y$ is even, then x is even or y is even.

a. State the contrapositive of the statement above. (Hint: Try to write out just "not(x is even or y is even)" first.)

Solution: The contrapositive is

If not(x is even or y is even), then not($x \cdot y$ is even).

The right part changes to "$x \cdot y$ is odd", but the left part is a little knotty, so we'll work on it separately.

$$\begin{aligned} \text{not}(x \text{ is even or } y \text{ is even}) &= \text{not}(x \text{ is even) and not}(y \text{ is even}) \\ &= x \text{ is odd and } y \text{ is odd} \end{aligned}$$

So, the contrapositive should be

If x is odd and y is odd, then $x \cdot y$ is odd.

b. Prove the original statement by proving the contrapositive.

Solution: With the contrapositive in hand, the statement is easy to prove. If x and y are both odd, then neither has a factor of 2 (that's what it means to be odd). This means that $x \cdot y$ doesn't have a factor of 2 either, so $x \cdot y$ must also be odd. So, the contrapositive is true. Since the contrapositive always has the same truth value as the original statement, the original statement must also be true.