University of South Carolina
Math 221: Math for Elementary Educators
Instructor: Austin Mohr
Section 001
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Test 2

1. Draw a picture to solve each problem using the specified method.
a. $3 - {}^{-}2$, set model

Solution: We have three pluses

$$+ + +$$

and we want to take away two minuses. Since there are no minuses to take away, we have to introduce them. For every minus we introduce, we also include a plus, so that the net change is zero.

$$+ + + - + - +$$

Notice that the picture above still represents 3. Now that there are minuses in the picture, we can take two of them away like the original problem asks.

$$+ + + + +$$

This picture is of 5, so $3 - {}^{-}2 = 5$.

b. ${}^{-}6 \div 2$, partition model

Solution: The problem says we have six minuses

$$- - - - - -$$

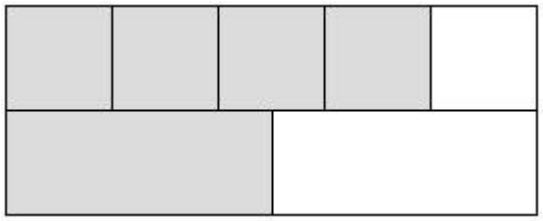
and we want to put them into two groups. This makes the picture

$$- - - \mid - - -$$

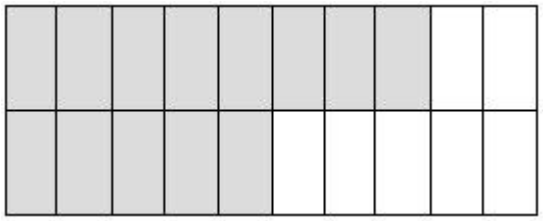
Each group has three minuses, so ${}^{-}6 \div 2 = {}^{-}3$

c. $\frac{4}{5} - \frac{1}{2}$, fraction tiles

Solution: The picture looks like



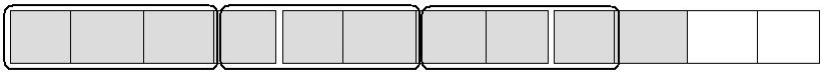
We want to know how much larger the top row is than the bottom row. They can't be compared directly - we need to break all the pieces into a common size. After playing around with fraction tiles (or thinking about common denominators), we see that tiles of size $\frac{1}{10}$ work.



Now it's easy to see that there are three more $\frac{1}{10}$ pieces in the top row than in the bottom row. This means that $\frac{4}{5} - \frac{1}{2} = \frac{3}{10}$.

d. $\frac{5}{2} \div \frac{3}{4}$, fraction tiles

Solution: Imagine we have $\frac{5}{2}$ (or $2\frac{1}{2}$) yards of fabric and it takes $\frac{3}{4}$ yards to make a single shirt. We get a picture like this



where each circle is one shirt. So, we have three complete shirts and some part of a shirt. What fraction of a shirt is $\frac{1}{4}$ yard of fabric? Since it takes three of these $\frac{1}{4}$ yard pieces to make a shirt, just having one of them makes $\frac{1}{3}$ of a shirt. So, $\frac{5}{2} \div \frac{3}{4} = 3\frac{1}{3}$ (or $\frac{10}{3}$).

2. Tell, without dividing, whether 3,675 is divisible by each of the following numbers. How do you know?

a. 2

Solution: 3,675 is not even, so it is not divisible by 2.

b. 3

Solution: The sume of the digits of 3,675 is 21, which is divisible by 3. So, 3,675 is divisible by 3.

c. 5
Solution: 3,675 ends in a 5, so it is divisible by 5.

d. 6
Solution: 3,675 is not divisible by 2, so it is not divisible by 6, since $6 = 2 \cdot 3$.

e. 15
Solution: 3,675 is divisible by both 3 and 5, so it is divisible by 15, since $15 = 3 \cdot 5$.

3. For the following questions, use the fact that

$$12,936 = 2^3 \cdot 3 \cdot 7^2 \cdot 11$$

$$44,100 = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2.$$

a. Find the greatest common divisor of 12,936 and 44,100.
Solution: Build the greatest common divisor by choosing the smaller exponent of each prime in factorizations of 12,936 and 44,100. Following this procedure, we get $2^2 \cdot 3 \cdot 5^0 \cdot 7^2 \cdot 11^0 = 588$.

b. Find the least common multiple of 12,936 and 44,100.
Solution: Build the least common multiple by choosing the larger exponent of each prime in factorizations of 12,936 and 44,100. Following this procedure, we get $2^3 \cdot 3^2 \cdot 5^2 \cdot 7^2 \cdot 11 = 970,200$.

c. Reduce the fraction $\frac{12936}{44100}$.
Solution: If we use the factorizations of the numerator and the denominator, we can easily see what numbers to cancel.

$$\begin{aligned} \frac{12936}{44100} &= \frac{2^3 \cdot 3 \cdot 7^2 \cdot 11}{2^2 \cdot 3^2 \cdot 5^2 \cdot 7^2} \\ &= \frac{2 \cdot 11}{3 \cdot 5^2} \\ &= \frac{22}{75}. \end{aligned}$$

4. What has to be true about a and b to make each statement true?

a. $\frac{a}{5} > \frac{b}{5}$

Solution: Since the denominator is 5 in both cases, it means the the fraction tiles on the left side and the right side are the same size. So, for the left side to be bigger, we need more pieces. That means we want $a > b$.

b. $\frac{3}{a} > \frac{3}{b}$

Solution: Since the numerator is 3 in both cases, it means that we have the same number of pieces on the left as we do on the right, but they might be of different sizes. If we want the left side to be bigger, then we want bigger pieces. To get bigger pieces, we want to break the unit tiles into fewer pieces. That means we want $a < b$.

c. $\frac{a}{2} = \frac{b}{6}$

Solution: To get from the fraction on the left to the fraction on the right, we multiplied the denominator by 3. If we want to keep them equal, we have to multiply the numerator by 3, as well. This means we want $b = 3a$.

5. Explain with words and/or pictures why each statement is true. (Simply saying that they are both the same number is not an explanation.)

a. adding a negative is the same as subtracting a positive

Solution: Think of the numbers in terms of money. Adding a negative value to a certain amount of money is adding a debt, but this is the same thing as taking away money (i.e. subtracting a positive value).

b. negative times negative is positive

Solution: Think of the numbers in terms of driving a car. Multiplying two negative numbers is like driving with negative speed and negative time, but this looks just like driving forward (i.e. covering positive distance).