

**University of South Carolina**  
**Math 221: Math for Elementary Educators**  
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**Section 001**  
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**Test 1**

Help Prime Minister Mittens govern Talking Cat Island by analyzing the following three word problems. For each problem

- a. tell what operation is involved ( $+$ ,  $-$ ,  $\times$ , or  $\div$ ),
- b. give the name of an appropriate model for the problem, and
- c. draw a picture demonstrating the solution.

1. Talking Cat Island is home to 24 thousand cats. If a district contains 3 thousand cats, how many districts are required? (Hint: Work with the numbers 24 and 3, not 24,000 and 3,000.)

Solution: We are given the total number of objects and the desired size of each group. We want to know how many groups are required. This indicates the repeated subtraction model of division.

2. Every second, 6 saucers of milk are consumed by the citizens of the island. How many saucers do they drink in 5 seconds?

Solution: The problem above involves multiplication. One could make a case for either repeated addition (each second, we get another group of 6) or the array model (each row contains 6 saucers, and we have a row representing each second). Personally, I would argue that the repeated addition model lends itself more naturally to the linear nature of time (6 saucers now, then 6 more, then 6 more, . . .), whereas the array structure seems arbitrarily imposed.

3. It takes one crew of catstruction workers 14 hours to build a scratching post, while it takes another crew only 11 hours to build the same post. How much longer does the first crew require than the second?

Solution: Whenever time is the thing we are actually interested in measuring (as opposed to the previous problem, where it simply helped us define our groups), the number line model tends to work best. Specifically, the

number line model of subtraction is appropriate here.

[2] 4. Describe the following sets of numbers.

a. Natural Numbers

Solution:  $\{1, 2, 3, \dots\}$

b. Whole Numbers

Solution:  $\{0, 1, 2, 3, \dots\}$

c. Integers

Solution:  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

d. Rational Numbers

Solution:  $\{\frac{a}{b} \mid a, b \text{ integers, } b \neq 0\}$

Loosely speaking, this says the rationals are all fractions you can make with numerator and denominator from the integers with the exception that the denominator is not allowed to be 0. Of course, many of these fractions will be equivalent (for example,  $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$ , etc.).

Another equally valid description would be to say that the rationals are those numbers whose decimal representations either terminate (such as 1.5) or repeat (such as  $2.\overline{3}$ ). Even this description has some redundancy, however (remember  $\overline{9} = 1$ ).

e. Real Numbers

Solution: Any number that can be written as a decimal (including non-terminating, non-repeating ones) is a real number. What kinds of numbers aren't real numbers? The next step up is the complex numbers, which incorporates things like  $\sqrt{-1}$  that don't make any sense in the reals.

5a. Convert  $342_5$  to base 10.

Solution: The number  $342_5$  indicates 3 flats, 4 sticks, and 2 singles of the base 5 variety. Each flat gives 25 singles and each stick gives 5 singles. So, we have a total of  $3 \times 25 + 4 \times 5 + 2 = 97$  singles. In other words,  $342_5 = 97_{10}$ .

5b. Convert  $113_{10}$  to base 5.

Solution: The number  $113_{10}$  can be viewed as 113 singles (here, I use 113 to really mean the "one hundred thirteen" of base 10). We can regroup this into base 5 blocks using 4 flats (which uses up 100 singles), 2 sticks (which





b. Why do we say that, for example,  $0 \div 1 = 0$  (and so is defined) but  $1 \div 0$  is undefined?

Solution: Let's consider these problems in the context of the repeated subtraction model. In  $0 \div 1$ , we have 0 objects and we want to know how many groups of size 1 we can make. Obviously, we can't make any groups (we don't even have any objects to put into groups), so our answer should be 0. On the other hand,  $1 \div 0$  means we have one object and we want to know how many groups of size 0 we can make. Somehow, this question doesn't even make sense. What is a group of size 0? What would limit us from making as many as we want? No matter how many we make, we'll always have that 1 object remaining (since we never actually put it in a group). No matter how hard we try, we can't find a numerical answer that makes any sense, so we just say that the answer is undefined.

c. Prove with a picture that  $(x + y)^2 = x^2 + 2xy + y^2$ . (Hint: Remember that  $(x + y)^2$  is the same as  $(x + y)(x + y)$ . Now, think about the area model of multiplication to make your picture.)

Solution: The picture below is a square whose sides are all length  $x + y$ . One way to express the area is  $(x + y)(x + y)$ , or  $(x + y)^2$ . Another way is to add up the smaller boxes that comprise the larger box. This gives  $x^2 + 2xy + y^2$ , so  $(x + y)^2 = x^2 + 2xy + y^2$ .





