University of South Carolina
Math 221: Math for Elementary Educators
Instructor: Austin Mohr
Section 001
Spring 2010

Quiz 6

1. Let $A = \{\text{all natural numbers divisible by } 2\}$ and $B = \{\text{all natural numbers divisible by } 3\}$ with universal set $U = \{\text{all natural numbers}\}$.

a. Is $A \subseteq B$? Why or why not?

Solution: No. There are plenty of numbers in A that are not in B. To find some, just think about numbers that are divisible by 2 but not by 3. Examples are $2, 4, 8, 10, 14, 16, \ldots$.

b. Describe $A \cap B$.

Solution: $A \cap B$ contains all the natural numbers that are divisible by 2 *and* are divisible by 3. More succinctly, $A \cap B = \{\text{all natural numbers divisible by } 6\}$.

c. Describe $B - A$.

Solution: $B - A$ contains all the natural numbers that are divisible by 3 but are *not* divisible by 2. More succinctly, $B - A = \{\text{all odd natural numbers divisible by } 3\}$.

d. Describe $\overline{A}$.

Solution: $\overline{A}$ contains all the natural numbers that are *not* divisible by 2 (remember, $\overline{A} = U - A$). More succinctly, $\overline{A} = \{\text{all odd natural numbers}\}$.

2. Your instructor has had far too much to drink and is disgracing himself by shouting mathematical falsehoods. Sober him up by stating the negation of each claim and proving that the negation is true.

a. Every integer is negative.

Solution: The negation is "Some intgers are not negative", and you might give examples like 1, 2, 3, ...

b. If a number ends in 3, then it is divisible by 3.

Solution: The negation is "Some numbers end in 3, yet they are not divisible by 3", and you might give examples like 13 or 23.

3. Prove the following statement by proving its contrapositive.

If x^2 is odd, then x is odd.
Solution: The statement

"If x is not odd, then x^2 is not odd."

is a perfectly correct contrapositive, but it's a little more natural (and useful) to say

"If x is even, then x^2 is even."

Now, x being even means $x = 2y$ for some other number y (we're just saying that x has at least one factor of 2). This means that

$$\begin{aligned} x^2 &= (2y)^2 \\ &= (2y) \cdot (2y) \\ &= 4y^2, \end{aligned}$$

and so we see that x^2 also has a factor of 2 (in fact, it has at least two factors of 2 because of the 4). This means that x^2 is also even, which is what we wanted to prove. Since the contrapositive is true, we know that the original statement is true, too.