**University of South Carolina**
**Math 221: Math for Elementary Educators**
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**Section 001**
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## Quiz 3

1. Draw a picture representing the solution of each problem using the specified model.
   a. $^{-}2 - 3$, set model

Solution: The problem says you have two minuses and you want to take away 3 pluses. As it stands, you can't take away pluses, because you don't have any. Since pluses and minuses cancel out, we can represent the 2 minuses we do have in a clever way.

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The picture above is still really just 2 minuses since, if we wanted, we could cancel the other pluses and minuses out. We don't want to do that though, since the whole point was to get some pluses in there to *take away*, not cancel. Now that we have some pluses, we can take away the 3 pluses the problem wanted us to. That leaves us with

$$- - - - -$$

so the answer is -5.

b. $^{-}2 \cdot {}^{-}3$, number line model

Solution: If we think about the car driving, this is asking "Right now, a car is sitting at the 0 mark on the number line. If it is driving -2 mph (i.e 2 mph to the left), where was it 3 hours ago?". One hour ago, it must have been at 2 on the number line. Two hours ago, it was at 4. Three hours ago, it was at 6.

c. $^{-}6 \div {}^{-}2$, repeated subtraction model (Hint: You don't need any "opposite group" nonsense to model this.)

Solution: For repeated subtraction, the problem says that we have 6 minuses and want to put them into groups that each contain 2 minuses. (In repeated subtraction, the divisor tells about the size of the groups. In partition, the divisor tells about the number of groups.) Start putting the 6 minuses into groups.

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Since we ended up with 3 groups, the answer is 3. Note that we didn't have to talk about opposite groups at all.

2. Number Theory Problems
   a. Determine <u>all</u> numbers dividing 105. (Hint: First, find the prime factorization of 105.)

Solution: Using a factor tree, you can determine that $105 = 3 \cdot 5 \cdot 7$. Any number that divides 105 has to be made up out of these prime numbers. To get the entire list, just write down every possible combination of them.

- 1 (1 divides *every* number, but we don't write it in a prime factorization since it isn't considered to be prime)
- 3
- 5
- 7
- 15 ($= 3 \cdot 5$)
- 21 ($= 3 \cdot 7$)
- 35 ($= 5 \cdot 7$)
- 105 ($= 3 \cdot 5 \cdot 7$)

b. Determine the least common multiple and greatest common divisor of 105 and 252.

Solution: Before we even start, we need the prime factorizations of 105 and 252.

$$105 = 3 \cdot 5 \cdot 7$$
$$252 = 2^2 \cdot 3^2 \cdot 7$$

Now, stop and think about the the problem is asking for. The least common multiple is the smallest number that is a multiple of both 105 and 252. In other words, it's the smallest number that both 105 and 252 can divide. Let's build the LCM out of primes. How many 2's should it have? 105 has no 2's, but 252 has two 2's. This means that the 105 doesn't require any 2's to be present in the LCM, but 252 needs two 2's in the LCM (if there weren't, how could 252 possibly divide it?). What about 3's? The 105 says "I require at least one 3" and the 252 says "I require at least two 3's". In order to satisfy them both, we have to go with two 3's. Continuing like this, we find out that the LCM needs to be built out of two 2's, two 3's, one 5, and one 7. So, the LCM of 105 and 252 is $2^2 \cdot 3^2 \cdot 5 \cdot 7 = 1260$. Notice that what we ended up doing to compute the LCM was to visit each prime in the 105 and the 252 and compare their exponents. We took the *bigger* exponent to be in the LCM.

The greatest common divisor is the biggest number that can divide both 105 and 252. We compute it in a similar way, but with one slight difference. For example, think about how many 2's should be in the GCD. The 105 has no 2's, so the GCD can't have any either (if it did, how could it possibly divide 105?). For the 3, the 105 says "The GCD can have at most one 3 in it" while the 252 says "The GCD can have at most two 3's in it". In order to satisfy them both, we have to go with only one 3. Continuing like this, the GCD of 105 and 252 is $3 \cdot 7 = 21$. Notice that what we ended up doing to compute the GCD was to visit each prime in the 105 and the 252 and compare their exponents. We took the *smaller* exponent to be in the GCD.