University of South Carolina
Math 221: Math for Elementary Educators
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Section 001
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4.4 - Prime and Composite Numbers

1. Make as many different sized rectangles as you can using 12 blocks. What are the factors of 12?

2. Make as many different sized rectangles as you can using 13 blocks. What are the factors of 13?

Writing a number as the product of smaller numbers is called a factorization. Numbers that can only be factorized as 1 times itself are called prime numbers. Numbers with more interesting factorizations are called composite numbers. For a technical reason, we don't consider 1 to be a prime number.

3. Sieve of Eratosthenes: Circle the 2 and cross out all larger numbers that are divisible by 2. The smallest number that won't be crossed out is 3, so circle the 3 and then cross out all larger numbers that are divisible by 3. Repeat until every number is either crossed out or circled.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What do all the circled numbers have in common? What do all the crossed out numbers have in common?

4. Use factor trees to write each of 360 and 882 as products of natural numbers that are as small as possible (but don't bother using 1's - they don't do anything anyway). What do you notice about the numbers you come up with?

The Fundamental Theorem of Arithmetic says that *every* natural number has a prime factorization. In other words, every number can be written as the product of only prime numbers.

5. One way to determine if a number is prime is to try dividing by all the natural numbers smaller than it (except for 1, of course).

If the answer ever comes out evenly, then the number must be composite. For example, 15 is composite because you $15 \div 3 = 5$. In other words, $15 = 3 \cdot 5$, so 15 has a non-boring factorization.

If the answer *never* comes out evenly, then the number must be prime. For example, 7 is prime because you can divide it by any number less than 7 (except for 1) and the answer will never come out evenly. In other words, it's only factorization is the boring factorization $1 \cdot 7$.

You can actually get away with much fewer checks, however. I claim that you can determine without a doubt that 101 is prime by performing only 4 divisions. What are these divisions and why are they sufficient? Can you state a more general rule (one that works for numbers other than 101)? (Hint 1: If 2 does not divide a number, can 4 possibly divide it? Hint 2: When do the numbers in your trial divisions become big enough that you don't need to bother checking them?)