University of South Carolina
Math 170: Finite Mathematics
Section 006
Spring 2012

Test 3

Please write *only* your name on the test sheet.
Place all work and answers on the blank sheets provided.

1. Roll two four-sided dice (the faces are labeled 1, 2, 3, and 4) and let X be the sum of the two faces. What is the probability distribution for X (i.e. the table of outcomes for X with their corresponding probabilities)?

   Solution: One can make the following table to summarize all the possible rolls of two four-sided dice. The row and column are the roll of the first and second die, respectively. The entry is the sum of the two dice.

	1	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

   So, the possible outcomes for X are the integers 2 through 8, inclusive. To determine the probability of each outcome, count the number of times it appears in the table and divide by 16.

x	2	3	4	5	6	7	8
$\mathrm{P}(X = x)$	1/16	2/16	3/16	4/16	3/16	2/16	1/16

2. You borrow \$10,000 at 9% interest and arrange to make monthly payments (so the interest is compounded monthly). What should your monthly payment be if you must pay off the entire loan (including interest) in exactly four years?

   Solution: We use the annuity formula for installment loans, since we may view it as the bank placing a large sum of money into an interest-bearing account (namely, you) and withdrawing a fixed amount every month. Plugging the available information into the annuity formula gives

   $$10,000 = PMT \cdot \frac{1 - (1 + 0.09/12)^{-4 \cdot 12}}{0.09/12}$$
   $$10,000 \approx PMT \cdot 40.185,$$

   and so PMT $\approx$ \$248.85 is the required monthly payment.

3. You are playing a game where you draw a single card at random from a standard fifty-two-card poker deck. You win or lose money based on the card you draw. Twenty-six of the cards result in a win of \$1. Twenty-two of the cards result in a win of \$2. Four

of the cards result in a loss of \$100. (There is no overlap in the sets of cards just described.) What is the expected value of this game?

Solution: Let X be the finite random variable describing your winnings. The random variable is 1 with probability 26/52, 2 with probability 22/52, and -100 with probability 4/52. So,

$$E(X) = 1 \cdot 26/52 + 2 \cdot 22/52 + (-100) \cdot 4/52 \approx -6.35,$$

which means you expect to lose \$6.35 each time you play.

4. You download a file to your computer three separate times. Each time, there is a 1% chance the file becomes corrupted during transmission. What is the probaility at least two of the three downloads are corrupted?

   Solution: Since there are only two possible outcomes (corrupted or not), this is a Bernoulli trial. Let X be the associated binomial random variable that counts the number of successes in this experiment. We want $\mathrm{P}(X \geq 2)$, which is equal to $\mathrm{P}(X = 2) + \mathrm{P}(X = 3)$. Using the formula for probability distribution of a binomial random variable, we get

   $$\mathrm{P}(X = 2) = 3\mathrm{C}2 \cdot 0.01^2 \cdot 0.99^1 \approx 0.000297$$

   and

   $$\mathrm{P}(X = 3) = 3\mathrm{C}3 \cdot 0.01^3 \cdot 0.99^0 \approx 0.000001.$$

   So, $\mathrm{P}(X \geq 2) = 0.000298$.

5. Stanford-Binet IQ scores are normally distributed with a mean of 100 and a standard deviation of 16. What percentage of the population has an IQ score between 104 and 112?

   Solution: Letting X denote a randomly chosen IQ score, we are asked to find $\mathrm{P}(104 \geq X \geq 112)$. Standardizing gives

   $$\mathrm{P}\left(\frac{104 - 100}{16} \leq Z \leq \frac{112 - 100}{16}\right),$$

   which simplifies to $\mathrm{P}(0.25 \leq Z \leq 0.75)$. In order to look this up in the table of values, we must write it as probabilities of the form $\mathrm{P}(0 \leq Z \leq b)$ for appropriate values of b. Since both values lie to the right of the mean, we want

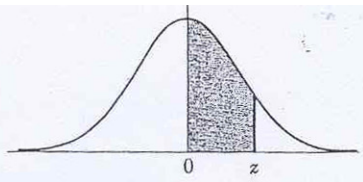
   $$\mathrm{P}(0.25 \leq Z \leq 0.75) = \mathrm{P}(0 \leq Z \leq 0.75) - \mathrm{P}(0 \leq Z \leq 0.25).$$

   (Draw a picture of the standard normal curve to see that we are overestimating the area with the first term and then subtracting the missing portion with the second.) Referencing the table, we get

   $$\mathrm{P}(0.25 \leq Z \leq 0.75) = 0.2734 - 0.0987 = .1747.$$

   So, roughly 17.47% of people have an IQ score between 104 and 112.

AREAS under the STANDARD NORMAL CURVE from 0 to z



z	0	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389