University of South Carolina
Math 170: Finite Mathematics
Section 006
Spring 2012

Test 2 Solutions

Please write *only* your name on the test sheet.
Place all work and answers on the blank sheets provided.

1. A student must choose whether to submit a writing assignment or a reading assignment (but not both). A writing assignment can be on any of three topics and must be either original research or a survey. For a reading assignment, the student must read one biography and one essay. There are three biographies and five essays from which to choose. How many different assignments are possible?

   Solution:
   Alternative 1: Choose a writing assignment.
   Step 1.1: Choose a topic. (3 choices)
   Step 1.2: Choose a format. (2 choices)

   Alternative 2: Chose a reading assignment.
   Step 2.1: Choose a biography. (3 choices)
   Step 2.2: Choose an essay. (5 choices)

   Total Choices = Alternative 1 + Alternative 2 = $(3 \cdot 2) + (3 \cdot 5) = 6 + 15 = 21$

2. Flip three indistinguishable coins simultaneously. What is the probability the result is two heads and one tail?

   Solution: If we treat the coins as distinguishable, there are $2^3 = 8$ total outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. Of these, three of them have exactly two heads and one tail (HHT, HTH, THH). So, the probability of that event is 3/8.

3. In a 1987 survey of married couples, 95% of all husbands were employed. Of all employed husbands, 71% of their wives were also employed. In every couple, at least one of the husband or wife was employed. What is the probability that the husband of an employed woman is also employed?

   Solution: Let H be the event that a husband is employed and W be the event that his wife is employed. We want the value of $\mathrm{P}(H \mid W)$, which is given by

   $$\frac{\mathrm{P}(H \cap W)}{\mathrm{P}(H \cap W) + \mathrm{P}(H' \cap W)}$$

   according to Bayes' Theorem. To find the needed values, we use

   $$\mathrm{P}(H \cap W) = \mathrm{P}(H)\,\mathrm{P}(W \mid H) = 0.95 \cdot 0.71 = 0.6745$$

   and

   $$\mathrm{P}(H' \cap W) = \mathrm{P}(H')\,\mathrm{P}(W \mid H') = 0.05 \cdot 1 = 0.05$$

(notice P($W \mid H'$) $= 1$ since, if the husband is not employed, the woman *must* be employed). Finally, we get

$$P(H \mid W) = \frac{0.6745}{0.6745 + 0.05} \approx 0.93.$$

So, if a woman is employed, there is roughly a 93% chance her husband is also employed.

4. You have six tiles labeled A, B, C, D, E, and F. What is the probability that a random four-letter word made from these tiles does not begin with "AB"? (Hint: The complementary event is easier to analyze.)

   Solution: Notice order matters when talking about words, so we'll want to use permutations rather than combinations. There are 4P2 four-letter words that begin with "AB", since the only freedom we have is to place the letters C, D, E, or F in the two remaining positions (the first two are taken by A and B, so the word looks like "A B _ _"). There are 6P4 words in total, since we have six letters to use and four positions to put them in. So,

   $$\text{P(Word begins with “AB”)} = \frac{\text{4P2}}{\text{6P4}} = \frac{1}{30}.$$

   Since P(Word begins with "AB") + P(Word not beginning with "AB") = 1, we get

   $$\text{P(Word not beginning with “AB”)} = 1 - \frac{1}{30} = \frac{29}{30} \approx .97.$$

   So, there is roughly a 97% chance that a randomly formed four-letter word does not begin with "AB".

5. A bag contains six red marbles, five blue marbles, and four yellow marbles. (All marbles are distinguishable.) How many sets of five marbles contain exactly one red marble or exactly two blue marbles?

   Solution: Let R be the collection of all five-marble sets containing exactly one red marble and B be the collection of all five-marble sets containing exactly two blue marbles. We want to find $n(R \cup B)$, which is equal to $n(R) + n(B) - n(R \cap B)$ by the addition principle.

   For $n(R)$, we can select a single red marble in 6C1 ways. After that, we choose any four non-red marbles in 9C4 ways. So, $n(R) = (6C1)(9C4) = 756$.

   For $n(B)$, we can select two blue marbles in 5C2 ways. After that, we choose any four non-red marbles in 10C3 ways. So, $n(B) = (5C2)(10C3) = 1200$.

   For $n(R \cap B)$, we select the red marble and blue marbles as before. The remaining two marbles cannot be red and cannot be blue, so we have 4C2 ways to select them. So, $n(R \cap B) = (6C1)(5C2)(4C2) = 360$.

   Plugging everything into the addition principle, we get $n(R \cup B) = 756 + 1200 - 360 = 1596$.

6. Prove the following statement. (Hint: Try a proof by contrapositive.)
   If $xy + x$ is even, then x is even or y is odd.

   Solution: The contrapositive of the statement above is:

   If not(x is even or y is odd), then not($xy + x$ is even).

A more natural way to state this (using DeMorgan's Law and the fact that even and odd are opposites) is:

If x is odd and y is even, then $xy + x$ is odd.

Our goal is to start with the fact that x is odd and y is even and conclude $xy + x$ is odd.

Since x is odd, we know $x = 2a + 1$ for some integer a (we don't know or care what a is). Since y is even, we know $y = 2b$ for some integer b (we don't know or care what b is). If we plug these expressions for x and y into $xy + x$, we get

$$\begin{aligned} xy + x &= (2a + 1)(2b) + (2a + 1) \\ &= 4ab + 2b + 2a + 1 \\ &= 2(2ab + a + b) + 1. \end{aligned}$$

The last line shows $xy + x$ can be written as "two times something plus one", which means $xy + x$ is odd.