University of South Carolina
Math 170: Finite Mathematics
Section 006
Spring 2012

Test 1 Solutions

Please write *only* your name on the test sheet.
Place all work and answers on the blank sheets provided.

1. A factory can produce 100 bicycles in a day at a total cost of \$10,500, and it can produce 120 bicycles in a day at a total cost of \$11,000.

   (a) Assuming a linear relationship, write a function $c(x)$ that gives the cost to manufacture x bicycles.

   Solution: Since the relationship is linear, we know $c(x) = mx + b$. The slope is given by $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11000-10500}{120-100} = 25$. That means $c(x) = 25x + b$. To find b, just plug in any point: $10500 = 25(100) + b$, so $b = 8000$. That means $c(x) = 25x + 8000$, and we're finished.

   (b) According to your function from part a, how many bicycles can be produced for \$20,000?

   Solution: Plugging in the information we know, we get $20000 = 25x+8000$, so $x = 480$. That means 480 bikes can be manufactured for \$20,000. (Notice we plugged in 20,000 for $c(x)$, not x. The variable $c(x)$ stands for cost, while x stands for the number of bikes.)

2. Let p be the price, in dollars, for a monorail ticket. Demand for tickets is given by the function $D(p) = 64p^{-0.76}$ (in thousands of tickets). The monorail company can provide service according to the function $S(p) = 2.5p + 15.5$ (in thousands of tickets).

   (a) What is the equilibrium price for the tickets? How many tickets will be sold at this price?

   Solution: We want to know at what price the two functions are equal. That is, we need to solve $64p^{-.076} = 2.5p + 15.5$ for p. This is basically impossible by hand, but graphing both functions and using the ISECT command on a TI-83 gives the intersection as approximately $(3.56, 24.38)$. That means the equilibrium price is \$3.56 and 24,380 tickets will be sold at that price.

   (b) What is the estimated shortage/surplus if the price is set at \$5 per ticket?

   Solution: Plugging \$5 into each equation gives $D(5) \approx 18.83$ and $S(5) = 28$. Since the supply is higher than the demand, there is a surplus of $28 - 18.83 = 9.17$ thousand tickets (i.e. 9,170 tickets) if the price is set at \$5.

3. A certain farm may grow soybeans, corn, and wheat. Set up but **do not solve** a linear programming problem to determine how many acres of each crop to plant in order to maximize its profit.

   - The farm encompasses 900 acres of land.
   - Fertilizer costs (per acre) are \$5 for soybeans, \$2 for corn, and \$1 for wheat.
   - Weekly labor costs (per acre) are 5 hours for soybeans, 2 hours for corn, and 2 hours for wheat.
   - Profits (per acre) are \$3,000 for soybeans, \$2,000 for corn, and \$1,000 for wheat.
   - You cannot spend more than \$3,000 for fertilizer.

- You must spend at least 2,000 hours of labor per week.

Solution: Maximize $3000x + 2000y + 1000z$ (profit) subject to

$$\begin{aligned} 5x + 2y + z &\le 3000 && \text{(fertilizer costs)} \\ 5x + 2y + 2z &\ge 2000 && \text{(labor costs)} \\ x + y + z &\le 900 && \text{(land usage)} \\ x \ge 0, y \ge 0, z &\ge 0 && \text{(only want positive solutions)} \end{aligned}$$

4. An electronics company acquires a warehouse full of cogs, sprockets, and widgets.

   - A Device requires 2 cogs, 1 sprocket, and 2 widgets.
   - A Gadget requires 1 cog, 1 sprocket, and 2 widgets.
   - A Thing requires 1 cog, 2 sprockets, and 1 widget.

   If the warehouse contains 350 cogs, 350 sprockets, and 400 widgets, how many Devices, Gadgets, and Things should the company produce to use up the entire inventory?

   Solution: It helps to write the system of equations first, then write the appropriate matrix. The variables over which we have control are d, g, t (the number of Devices, Gadgets, and Things we produce) and the resource we're given is the fixed number of cogs, sprockets, and widgets. This allows us to build the system

$$\begin{aligned} 2d + g + t &= 350 \\ d + g + 2t &= 350 \\ 2d + 2g + t &= 400. \end{aligned}$$

   The corresponding matrix equation is obtained by just writing down everything in the order in which it appears in the system.

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} d \\ g \\ t \end{bmatrix} = \begin{bmatrix} 350 \\ 350 \\ 400 \end{bmatrix}$$

   If we call the matrix equation above $AX = B$, then solving for X gives

$$X = A^{-1}B = \begin{bmatrix} 100 \\ 50 \\ 100 \end{bmatrix}.$$

   So, in order to use up the entire inventory, the company should produce 100 Devices, 50 Gadgets, and 100 Things.

5. Consider a lopsided version of the Odds and Evens game: Each player may show either one or two fingers. If the sum is odd, you win \$1. If the sum is even, you lose \$2. What is your safest strategy according to the minimax criterion?

   Solution: According to the minimax criterion, we only need to defend against pure strategies by the opponent. We'll start with the generic strategy matrix for the row player and see how it fares against the opponent's pure strategies.

$$\begin{bmatrix} x & 1-x \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = [-3x + 1]$$

$$\begin{bmatrix} x & 1-x \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = [3x - 2]$$

   So, the minimax strategy is to choose the x for which $-3x+1 = 3x-2$, which is $x = 1/2$. In words, it means we should show an odd number of fingers half the time and an even number of fingers half the time.

6. A small town has a lumber mill and a paper factory. In a particular month, the lumber mill used \$30,000 of its own resources and \$100 of the paper factory's resources to produce a total of \$100,000 worth of goods. In the same month, the paper factory used \$20,000 of its own resources and \$7,000 of the lumber mill's resources to produce a total of \$200,000 worth of goods. What monthly production levels are necessary to meet an external demand of \$200,000 in lumber products and \$300,000 in paper products?

   Solution: *Carefully* form the technology matrix A (it helps to label the rows and columns so you know where materials are coming from and where they are going to).

$$\begin{array}{c} \\ \text{from paper} \\ \text{from lumber} \end{array} \begin{array}{c} \begin{array}{cc} \text{to paper} & \text{to lumber} \end{array} \\ \begin{bmatrix} 20000/200000 & 100/100000 \\ 7000/200000 & 30000/100000 \end{bmatrix} \end{array}$$

   The demand matrix D is

$$\begin{bmatrix} 300000 \\ 200000 \end{bmatrix}$$

   (notice paper comes before lumber, since that's the order I chose before). The production matrix is given by the formula $X = (I - A)^{-1}D$, which works out to roughly

$$\begin{bmatrix} 333669 \\ 302397 \end{bmatrix}$$

   So, approximately \$334,000 in paper products and \$302,000 in lumber products are needed to meet the demand.