**University of South Carolina**
**Math 170: Finite Mathematics**
**Section 006**
**Spring 2012**

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## Linear Programming

Loosely speaking, a *linear programming problem* is a problem in which we are to maximize or minimize a linear function (called the *objective function*) while simultaneously satisfying a number of linear inequalities (called *constraints*). The book approaches these problems with the simplex method. Like Gauss-Jordan, it is time-consuming and error-prone. Instead, we will trust WolframAlpha's sorcery.

**Problem 21** (page 286 in the text, paraphrased)
Your factory makes vanilla and mocha ice cream.
Each quart of vanilla requires 2 eggs and 3 cups of cream.
Each quart of of mocha requires 1 egg and 3 cups of cream.
You have in stock 500 eggs and 900 cups of cream.
You make \$3 profit for each quart of vanilla.
You make \$2 profit for each quart of mocha (because, seriously, mocha ice cream?).
How many quarts of each flavor should you make to earn the greatest profit?

**Solution**
Let's identify our variables first. The thing we have control over is the number of quarts of each flavor we produce. So, let

$$v = \text{number of quarts of vanilla produced}$$
$$m = \text{number of quarts of mocha produced.}$$

Let's identify the objective function next. The thing we want to maximize is profit, which is given by

$$3v + 2m \text{ (in dollars).}$$

The constraints involve the number of eggs and cups of cream we have in stock. We can't use more than 500 eggs. A quart vanilla uses 2 eggs, and a quart of mocha uses 1 egg. So, we get the constraint

$$2v + m \leq 500.$$

The information about cream gives the constraint

$$3v + 3m \leq 900.$$

Notice, in both cases, we did *not* write "equals". Doing that would force us to use *all* of the eggs or *all* of the cream, respectively. We aren't interested in using our whole inventory unless we have to. If we get a higher profit without using the whole inventory, we're probably happier in most cases, anyway.

There are two other constraints that you should include in basically every problem of this kind:

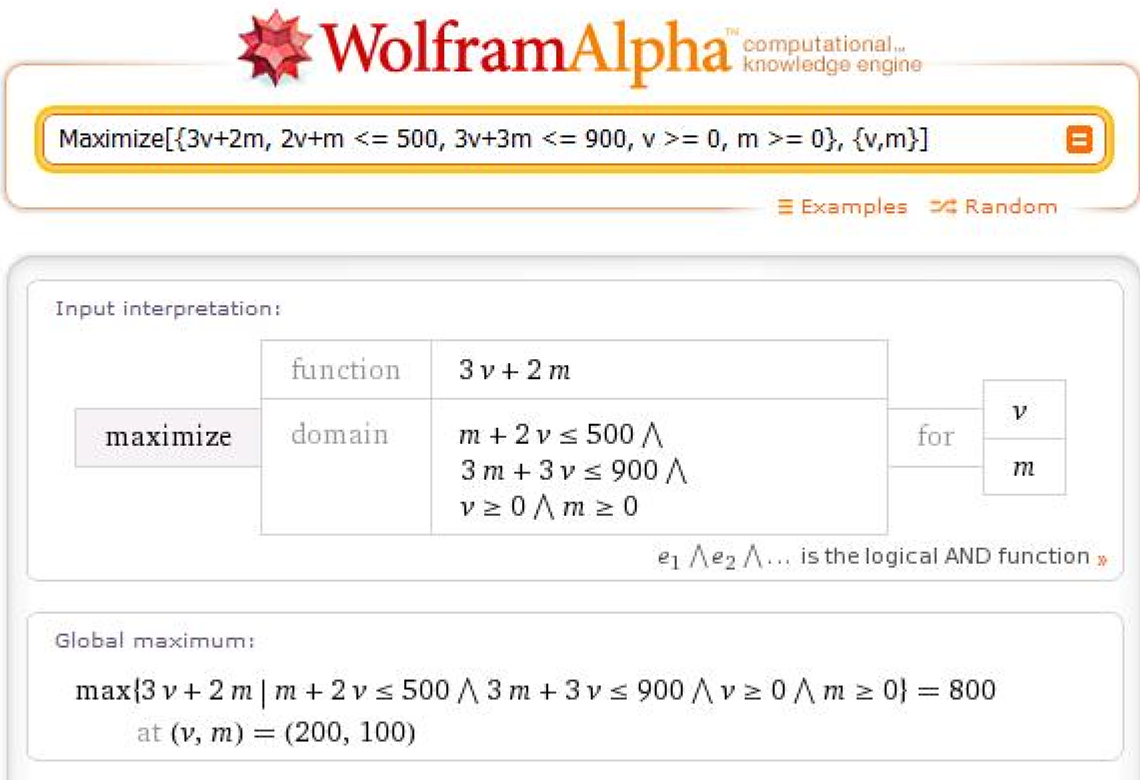
$$v \geq 0$$
$$m \geq 0$$

This prevents a solution involving negative quarts of ice cream, which obviously doesn't make sense in context but might have shown up mathematically if we didn't include these constraints.

Summarizing everything, we get the linear programming problem.

$$\begin{aligned} \text{Maximize: } & 3v + 2m \\ \text{Subject to: } & 2v + m \leq 500 \\ & 3v + 3m \leq 900 \\ & v \geq 0 \\ & m \geq 0 \end{aligned}$$

Once we have all this formulated, we let WolframAlpha take over.




The input to the Maximize function requires the objective function first, followed by all the constraints. The "{v,m}" at the end is just us telling WolframAlpha which letters are our variables. (In more advanced linear programming, there could be letters that don't actually represent variables. We won't be doing anything like that in this course.)

The output says that the maximum profit is $800 and is achieved by producing 200 quarts of vanilla and 100 quarts of mocha.

**Some Notes**

- If you want to minimize the objective function instead, replace "Maximize" with "Minimize".
- In your constraints, type "=" for "equals", "<=" for "less than or equal to", and ">=" for "greater than or equal to".
- Do not include the name of any of the functions in your input. In this example, we might have said something like "the profit is $p = 3v + 2m$". The $p$ is shorthand for $p(v, m)$ and is a *dependent* variable; it depends on $v$ and $m$. We only want to include the *independent* variables $v$ and $m$ in our input to WolframAlpha.