University of South Carolina
Math 170: Finite Mathematics
Section 006
Spring 2012

Variable vs. Marginal

Every function can be seen as the fixed part (the part that doesn't depend on the input x) plus the variable part (the part that does depend on the input x).

For example, if $f(x) = 2x + 3$, then 3 is the fixed part and $2x$ is the variable part. As another example, if $g(x) = 2x^2 + 3x + 4$, then 4 is the fixed part and $2x^2 + 3x$ is the variable part.

If your function happens to represent cost/revenue/profit, then you call the fixed part the **fixed cost/revenue/profit** and you call the variable part the **variable cost/revenue/profit**.

Marginal cost/revenue/profit asks "If I've already produced x items, what's the additional cost/revenue/profit of producing one more item?"

For ease of communication, suppose f and g above are both cost functions. To find the marginal cost, you need to find the cost of producing $x + 1$ items and then subtract the cost of producing x items.

Taking $f(x)$ as an example, the marginal cost is given by

$$\begin{aligned} f(x+1) - f(x) &= (2(x+1)+3) - (2x+3) \\ &= (2x+5) - (2x+3) \\ &= 2. \end{aligned}$$

So, no matter how many items you've produced, the next one always increases your cost by exactly \$2. (For a linear function, it turns out the marginal cost is always equal to the slope. Do you see why?)

As another example, the marginal cost of $g(x)$ is given by

$$\begin{aligned} g(x+1) - g(x) &= (2(x+1)^2 + 3(x+1) + 4) - (2x^2 + 3x + 4) \\ &= (2x^2 + 7x + 9) - (2x^2 + 3x + 4) \\ &= 4x + 5. \end{aligned}$$

So, the more items you produce, the more the next one is going to cost. For example, producing your tenth item costs an extra $4 \cdot 10 + 5 = 45$ dollars, while producing your one hundredth item costs an extra $4 \cdot 100 + 5 = 405$ dollars.