

University of South Carolina  
Math 115: Precalculus  
Instructor: Austin Mohr  
Section 006  
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Test 3

Do not write on this page. Instead, use the blank paper provided to show all your work and answers. Credit will not be given if no work is shown.

1. Talking Cat Island

Your plane lands on-time at Talking Cat Island International Airport. Her Excellency the Prime Minister Mittens greets you with a warm nuzzle, as is the custom here. Getting straight to business, she presents you with data regarding the milk supply crisis on Talking Cat Island. Experts estimate the current milk supply (in thousands of gallons) to be given by  $f(t) = 350 \cdot .75^t$ , where  $t$  denotes the number of years from today.

- a. How many gallons of milk are currently possessed by the residents of Talking Cat Island?
- b. How many gallons will remain at this time next year?
- c. How many years will it take before the milk levels drop to less than 10,000 gallons? (Trying values until you get it right will not earn you credit. You must use a more sophisticated method.)

2. Logarithmic and Trigonometric Identities

- a. Contract as much as possible.

$$\frac{\frac{1}{2} \log w + \log x}{3 \log(y + 1) - \log y}$$

- b. Show that the following equation is an identity.

$$\frac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x} = \tan^2 x$$

- c. Show that the following equation is an identity.

$$\frac{\cos(-x)}{1 - \sin x} = \frac{1 - \sin(-x)}{\cos x}$$

### 3. Graphs of Trigonometric Functions

- Sketch one cycle of the graph of  $-2\sin\left(x + \frac{\pi}{2}\right)$ . Give the coordinates of one hill, one valley, and one x-intercept.
- Sketch one cycle of the graph of  $\tan(2x) + 1$ . Give the equations of the two asymptotes closest to your cycle and give the coordinates of the y-intercept.
- Give the equation of a cosine wave with period  $\frac{\pi}{4}$ , phase shift  $\frac{\pi}{12}$ , and range  $[2, 12]$ .

### 4. Logarithmic and Trigonometric Equations

- Solve for  $x$ . Your answer should be exact.

$$\log(10x + 9) - \log x = 2$$

- Solve for  $x$ . You may give either an exact or an approximate answer.

$$5^{x+1} = 3^x$$

- Find  $\sin(2\alpha)$  given that  $\sin(x) = \frac{2}{5}$  and  $\alpha$  is in quadrant II. Give an exact answer in radians.
- Find all values of  $x$  that satisfy. Give an exact answer in radians.

$$\sin^2(3x) = \sin(3x)$$

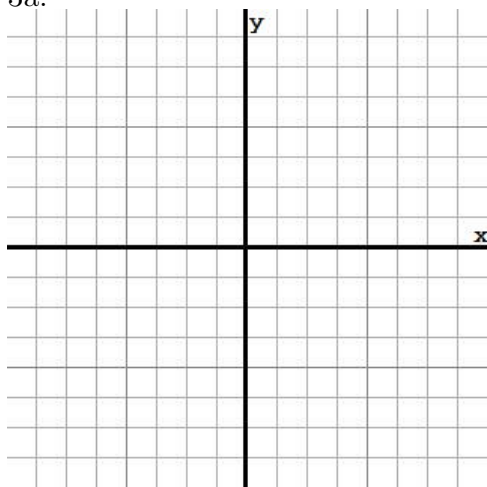
### Extra Credit

There are seven ways to arrange four labeled balls in two unlabeled buckets so that neither bucket is empty. Show all seven of these arrangements.

Answer Sheet

Use the axes provided to graph your solution to problems 2a and 2b.

3a.



3b.

