

University of South Carolina
Math 115: Precalculus
Instructor: Austin Mohr
Section 006
Fall 2009

Final Exam

Do not write on this page. Instead, use the blank paper provided to show all your work and answers. Credit will not be given if no work is shown.

1. Solve for x

In each part, find *all* values of x satisfying the given statement.

a. $2^{x-1} = 7^x$

b. $\sqrt{2x+3} + \sqrt{x} = 2$

c. $\log(x) + \log(x+3) = 1$ (Hint: There are two solutions, but one of them is no good.)

d. $\sin^2(5x) = \frac{1}{2} \sin(5x)$

e. $\frac{x^2+3x}{x-2} \geq 0$ (Notice this is an *inequality*, not an equation.)

2. Constructing Equations

a. Give the equation of a line through the points $(1, 2)$ and $(3, -3)$.

b. Give the equation of a circle with center $(1, 2)$ and through the point $(4, 10)$.

c. Give the equation of a parabola that has been flipped vertically, stretched by a factor of 2, shifted up 3 units, and shifted left 4 units.

d. Give the equation of $f^{-1}(x)$ if $f(x) = (2x+1)^{\frac{-1}{5}}$.

e. Give the equation of the sine wave with period $\frac{\pi}{12}$, phase shift $-\frac{\pi}{4}$, and range $[3, 9]$.

3. Algebraic Manipulation

a. Find $(f \circ g)(x)$ if $f(x) = x^2 - x$ and $g(x) = 2x + 1$.

b. Find $f(x+h) - f(x)$ if $f(x) = x^2 + 3x - 2$.

c. Simplify $\sqrt[3]{27x^3y^6} + \sqrt{\frac{4x^{-2}}{y^4}}$ as much as possible.

d. Contract $(\frac{1}{2} \log x - \log y)(3 \log(x+1) + \log y)$ as much as possible.

e. Show that $\frac{1}{\csc x - \cot x} = \frac{1+\cos x}{\sin x}$ is an identity.

f. Decompose $\frac{1}{x^2-x}$ into partial fractions.

g. Set up but do not solve the partial fraction decomposition for $\frac{3x^2+7x-4}{(x^2+1)(x-3)^3}$.

4. Graphs

- Find the domain of $f(x) = \sqrt{1 - 2x}$.
- Find the domain of $f(x) = \frac{x^2 - 1}{2x^2 + 4x - 30}$.
- Find the roots and asymptotes of $f(x) = \frac{x^2 - 16}{x^2 + 4x + 3}$.
- Find the roots (and their behavior) and end behavior of $f(x) = -3(x + 2)(x - 4)^2$.
- Sketch one cycle of $f(x) = 2 \cos(x - \frac{\pi}{2})$. Give the coordinates of the five key values (three roots, a hill, and a valley).
- Graph the solution of the system of inequalities.

$$\begin{aligned}y - 2x &< 0 \\ y &\geq x^2\end{aligned}$$

Extra Credit 1

Solve the system of equations. Your answer will not be unique.

$$\begin{aligned}x + 2y + 4z &= 3 \\ 2x - y - 2z &= 1 \\ 3x + 4y + 8z &= 7\end{aligned}$$

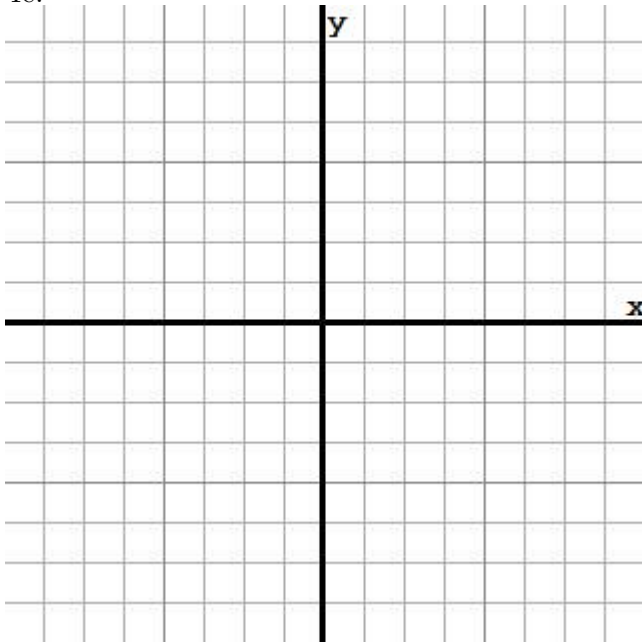
Extra Credit 2

Find the exact value of $\sin\left(\frac{\alpha}{2}\right)$ given that $\sin \alpha = \frac{3}{5}$ and $\frac{\pi}{2} < \alpha < \pi$.

Graph Sheet

Use the axes provided to graph your solution to problems 4e and 4f.

4e.



4f.

