University of South Carolina
Math 111i: Intensive College Algebra
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Section 1
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Test 3
(200 points total)

[25] 1. Contract $\frac{\frac{1}{3}log(w)+log(x)}{2log(y+1)-log(y)}$ as much as possible. The three important log rules here are

1. $log(AB) = log(A) + log(B)$
2. $log(\frac{A}{B}) = log(A) - log(B)$
3. $log(A^p) = p \cdot log(A)$

We try to find instances of the righthand sides of these equations and replace them with the lefthand sides so that we end up with fewer logs when we are done.

$$\frac{\frac{1}{3}log(w) + log(x)}{2log(y+1) - log(y)} = \frac{log(w^{\frac{1}{3}}) + log(x)}{log((y+1)^2) - log(y)} \qquad \text{(Rule 3)}$$

$$= \frac{log(w^{\frac{1}{3}}x)}{log((y+1)^2) - log(y)} \qquad \text{(Rule 1 - notice the } \frac{1}{3} \text{ is on } \textit{only} \text{ the } w\text{)}$$

$$= \frac{log(w^{\frac{1}{3}}x)}{log\left(\frac{(y+1)^2}{y}\right)} \qquad \text{(Rule 2 - notice the 2 is on } \textit{only} \text{ the } y+1\text{)}$$

There is no rule to contract $\frac{log(A)}{log(B)}$, so we have to stop here.

[25] 2. Solve for x if $log(10x + 9) - log(x) = 2$. The plan of attack here is to contract the logarithms, then cancel the log with $10^{log(x)} = x$.

$$log(10x + 9) - log(x) = 2$$

$$log(\frac{10x+9}{x}) = 2 \qquad \text{(Rule 2)}$$

$$10^{log(\frac{10x+9}{x})} = 10^2 \qquad (10^{LHS} = 10^{RHS})$$

$$\frac{10x+9}{x} = 100 \qquad (10^{log(x)} = x)$$

$$10x + 9 = 100x$$

$$9 = 90x$$

$$.1 = x$$

[30] 3. The growth of a population of 20 bacteria can be modeled by $p(t) = 20 \cdot 4^t$ where t is measured in minutes.

a. How many bacteria are present after 3 minutes?

We are given the time (the value for t) and we want to find the output. So, we substitute this value for t.

$$f(3) = 20 \cdot 4^3$$

$$= 1,280$$

b. How many minutes are required for the population to grow to 3000 bacteria? (Do not simply try values until it works. You must actually solve for something here.)
We are given the population size (the value for $f(t)$) and we want to find the input. So, we substitute this value for $f(t)$.

$$\begin{aligned} 3000 &= 20 \cdot 4^t \\ 150 &= 4^t \\ ln(150) &= ln(4^t) \\ ln(150) &= t \cdot ln(4) && \text{(Rule 3)} \\ \frac{ln(150)}{ln(4)} &= t \\ 3.64 &\approx t \end{aligned}$$

c. What is the doubling time for this population?
Doubling time is the time required for the population to double in size. In other words, we want to know when the population will grow to 40 bacteria. Once we realize this, the approach is exactly the same as in part (b).

$$\begin{aligned} 40 &= 20 \cdot 4^t \\ 2 &= 4^t \\ ln(2) &= ln(4^t) \\ ln(2) &= t \cdot ln(4) && \text{(Rule 3)} \\ \frac{ln(2)}{ln(4)} &= t \\ .5 &= t \end{aligned}$$

d. Rewrite $p(t)$ in the form $p(t) = Ce^{kt}$.
The equation is currently in Ca^t form, so to change it to Ce^{kt} form, we have to solve

$$a = e^k$$

We know that $a = 4$, so we just have to solve for k.

$$\begin{aligned} 4 &= e^k \\ ln(4) &= ln(e^k) \\ ln(4) &= k \\ 1.39 &\approx k \end{aligned}$$

[30] 4. A savings account has a 9% annual interest rate.
a. Construct a function $f(t)$ to model the growth of an initial investment of $1000 where t is measured in years and interest is compounded monthly.
The formula for interest compounded n times is

$$P(t) = P_0(1 + \frac{r}{n})^{nt}$$

where P_0 is the initial investment, r is the interest rate, and n is the number of compoundings per year. We just take the given values and plug them into the formula.

$$P(t) = 1000(1 + \frac{.09}{12})^{12t}$$

b. Determine the effective interest rate of the account in part (a).
To determine effective interest, we should first convert the equation to Ca^t form.

$$a = (1 + \frac{.09}{12})^{12}$$
$$\approx 1.0938$$

Since growth factor = 1 + growth rate, we have to drop the 1 to get the growth rate (same thing as interest rate). So, the effective interest rate is 9.38%.

c. Construct a function $g(t)$ to model the growth of an initial investment of \$1000 where t is measured in years and interest is compounded continuously.
The formula for interest compounded n times is

$$P(t) = P_0 e^{rt}$$

where P_0 is the initial investment and r is the interest rate. We just take the given values and plug them into the formula.

$$P(t) = 1000e^{.09t}$$

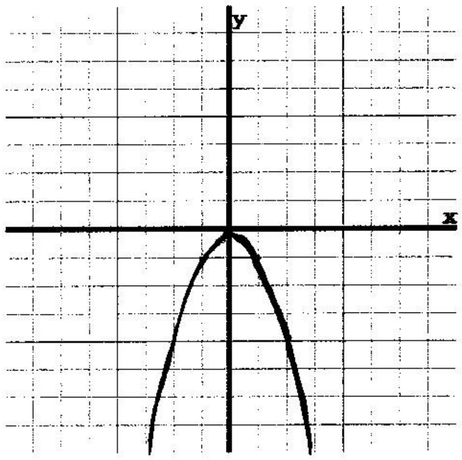
d. Determine the effective interest rate of the account in part (c). To determine effective interest, we should first convert the equation to Ca^t form.

$$a = e^{.09}$$
$$\approx 1.0942$$

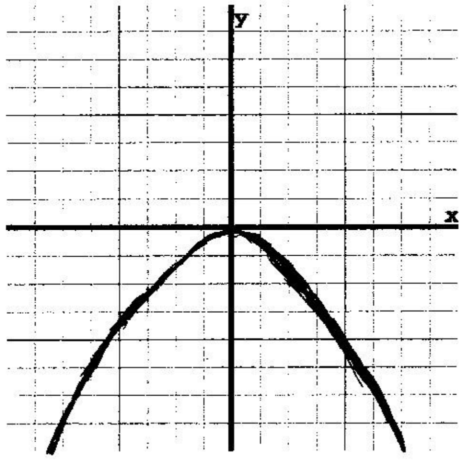
Since growth factor = 1 + growth rate, we have to drop the 1 to get the growth rate (same thing as interest rate). So, the effective interest rate is 9.42%.

[30] 5. Sketch the graph of $f(x) = -\frac{1}{3}(x+2)^2 + 6$ by performing the translations one at a time. At each step, write the function of the graph you are sketching.
The function is written in vertex form $(a(x-h)^2 + k)$, so we can identify the four translations taking place in the equation, then perform them one at a time.

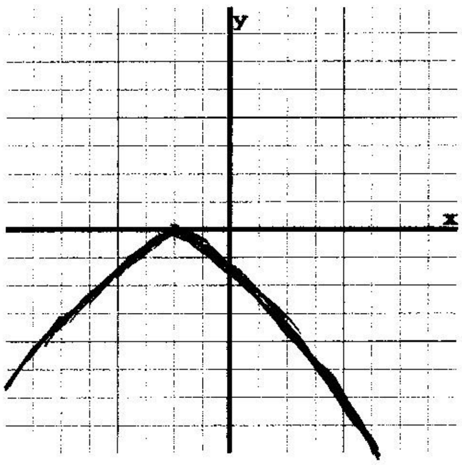
- The negative sign means vertical flip
- The $\frac{1}{3}$ means a vertical compression (since the value is between 0 and 1)
- The $(x+2)$ means a horizontal shift 2 units to the left (since this should be read as $(x-(-2))$)
- The +6 at the end means a vertical shift 6 units up



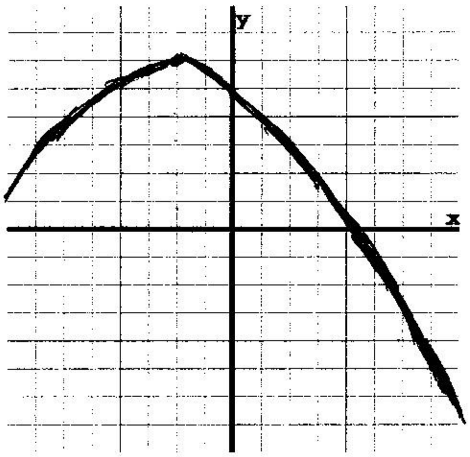
$-x^2$



$-\frac{1}{3}x^2$



$-\frac{1}{3}(x+2)^2$



$-\frac{1}{3}(x+2)^2 + 6$

[30] 6. Factor each expression and find the roots of the function.
Follow these steps whenever you need to factor.

1. Look for common factors, such as every coefficient being divisible by the same number or every term having factors of x.
2. If one of your factors is quadradtic $(x^2 + bx + c)$, list the factors of c.
3. Choose the pair of factors that add up to b.

Once you have the factored form of the quadratic, you can set each factor (each set of parenthesis) equal to 0 and solve them separately.

a. $f(x) = -4x^2 + 2x$

$$-4x^2 + 2x = 2x(-2x + 1) \qquad \text{(Common Factors)}$$

Neither of these factors is quadratic, so we can't factor any further. To find the roots, set each factor equal to 0.

$$\begin{aligned} 2x &= 0 \\ x &= 2 \end{aligned}$$

or

$$\begin{aligned} -2x + 1 &= 0 \\ 1 &= 2x \\ \frac{1}{2} &= x \end{aligned}$$

b. $g(x) = x^2 - 25$

There are no common factors, so we proceed to step 2. The factors of -25 are

- 1, -25
- -1, 25
- 5, -5

We want the pair that adds up to 0, which is 5, -5. So, the factored form is $(x+5)(x-5)$. To find the roots, set each factor equal to 0.

$$\begin{aligned} x + 5 &= 0 \\ x &= -5 \end{aligned}$$

or

$$\begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

c. $h(x) = 3x^2 + 12x - 36$

$$3x^2 + 12x - 36 = 3(x^2 + 4x - 12) \qquad \text{(Common Factors)}$$

We now focus only on the quadratic portion in the parentheses. The factors of -12 are

- 1, -12
- -1, 12

- 2, -6
- -2, 6
- 3, -4
- -3, 4

The pair that sums to 4 is -2, 6. So, the factored form is $3(x-2)(x+6)$. To find the roots, set each factor to 0 (except for the 3 - it doesn't have an x so it doesn't contribute anything).

$$\begin{aligned} x-2 &= 0 \\ x &= 2 \end{aligned}$$

or

$$\begin{aligned} x+6 &= 0 \\ x &= -6 \end{aligned}$$

[30] 7. A man stands on a ladder and throws a baseball. The height (in feet) of the baseball t seconds after being thrown is given by $h(t) = 13 + 40t - 16t^2$.

a. What is the initial height of the baseball (i.e. the height of the ball just before it is thrown)?
If the ball has not been thrown yet, then $t = 0$. To find the height when $t = 0$, we just substitute into the equation.

$$\begin{aligned} h(0) &= 13 + 40t - 16t^2 \\ &= 13 + 40 \cdot 0 - 16 \cdot 0^2 \\ &= 13 \end{aligned}$$

b. At what time does the baseball attain maximum height? What is the maximum height?
Maxima and minima always occur at the vertex. We have to find the vertex one coordinate at a time. Use the formula to find the t-coordinate

$$\begin{aligned} t &= \frac{-b}{2a} \\ &= \frac{-40}{2 \cdot (-16)} \\ &= 1.25 \end{aligned}$$

To find the corresponding h-coordinate, put this value for t into the function.

$$\begin{aligned} h(1.25) &= 13 + 40 \cdot 1.25 - 16 \cdot 1.25^2 \\ &= 38 \end{aligned}$$

So, the maximum height occurs at 1.25 seconds and the maximum height is 38 feet.

c. When does the baseball hit the ground?
If the baseball is on the ground, then the height is 0. So, we need to solve

$$0 = 13 + 40t - 16t^2$$

This will be too hard to factor, so we should use the quadratic formula instead. Remember, when using the quadratic formula, to work very slowly. If you try to type it all into your calculator at once, you probably won't get the right answer.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-40 \pm \sqrt{40^2 - 4 \cdot (-16) \cdot 13}}{2 \cdot (-16)} \\
&= \frac{-40 \pm \sqrt{1600 - (-832)}}{-32} \\
&= \frac{-40 \pm \sqrt{2432}}{-32} \\
&= \frac{-40 \pm \sqrt{2432}}{-32} \\
&\approx \frac{-40 \pm 49.32}{-32} \\
&= \frac{9.32}{-32} \text{ or } \frac{-89.32}{-32} \\
&= -.29 \text{ or } 2.79
\end{aligned}$$

The negative answer is meaningless in the context of the word problem, so we conclude that the ball hits the ground after 2.79 seconds.