University of South Carolina
Math 111i: Intensive College Algebra
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Section 1
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Test 2
(200 points total)

[15] 1. Convert 5 meters per minute to inches per day. Assume that 1 inch = 2.5 centimeters.

Remember to transform only one unit at a time. Here, we changed meters to inches first, then minutes to days.

$5\frac{\text{m}}{\text{min}} \cdot \frac{100\text{ cm}}{1\text{ m}} \cdot \frac{1\text{ in.}}{2.5\text{ cm}} = \frac{5 \cdot 100}{2.5} \frac{\text{in.}}{\text{min.}} = 200 \frac{\text{in.}}{\text{min.}}$
$200\frac{\text{in.}}{\text{min}} \cdot \frac{60\text{ min.}}{1\text{ hour}} \cdot \frac{24\text{ hour}}{1\text{ day}} = \frac{200 \cdot 60 \cdot 24}{1} \frac{\text{in.}}{\text{day}} = 288,000 \frac{\text{in.}}{\text{day}}$

[20] 2. Evaluate the expression or solve for x, as appropriate.

The two important rules of logs are:

- $log(10^x) = x$
- $10^{log(x)} = x$

We use the first rule directly in parts a and b by rewriting the number as a power of 10. In part c, we use the second rule by introducing a base 10 ($10^{LHS} = 10^{RHS}$). In part d, we use the first rule again by taking the log of both sides of the equation. Remember that *log* and "10" (that is, 10 with an exponent on it) cancel each other in the same way that square and square root cancel. For parts c and d, remember to cancel out the number in front (the 3 and the 5 in these examples) before you try to deal with the "10" or the log.

a. $log(100\ 000)$

$$\begin{aligned} log(100000) &= log(10^5) \\ &= 5 \end{aligned}$$

b. $log(.01)$

$$\begin{aligned} log(.01) &= log(10^{-2}) \\ &= -2 \end{aligned}$$

c. $3log(3x + 400) = 9$

$$\begin{aligned} 3log(3x + 400) &= 9 \\ log(3x + 400) &= 3 \\ 10^{log(3x+400)} &= 10^3 \\ 3x + 400 &= 1000 \\ 3x &= 600 \\ x &= 200 \end{aligned}$$

d. $5 \cdot 10^{2x+1} = 5000$

$$\begin{aligned}
5 \cdot 10^{2x+1} &= 5000 \\
10^{2x+1} &= 1000 \\
log(10^{2x+1}) &= log(1000) \\
2x+1 &= 3 \\
2x &= 2 \\
x &= 1
\end{aligned}$$

[30] 3. Write in simplest terms. Your answer should not contain negative exponents or radicals.

We had a bunch of exponent rules, but we eventually got rid of them in favor of a slightly more intuitive set. Our new list is something like

- Add exponents when combining: $x^2 \cdot x^3 = xx \cdot xxx = x^5$
- Multiply when raising a power to a power: $(x^2)^3 = (xx)^3 = (xx)(xx)(xx) = x^6$
- Cancel when on oposite sides of a fraction: $\frac{x^2}{x^3} = \frac{xx}{xxx} = \frac{1}{x}$
- "Distributive law": $(xy)^2 = (xy)(xy) = xxyy = x^2y^2$
- Passing through the fraction bar flips the sign of an exponent: $\frac{1}{x^{-2}} = x^2$ and $x^{-2} = \frac{1}{x^2}$
- Roots become fractional exponents: $\sqrt{x} = x^{\frac{1}{2}}$

Note that the last rule does <u>not</u> allow you to change, say, -64 into $\frac{1}{64}$. The rule only lets you get rid of negative signs that occur in the exponent.

a. $(\frac{-4x^3y^4}{x^4y})3$

$$\begin{aligned}
\left(\frac{-4x^3y^4}{x^4y}\right)^3 &= \left(\frac{-4y^3}{x}\right)^3 \\
&= \frac{(-4)^3(y^3)^3}{x^3} \\
&= \frac{-64y^9}{x^3}
\end{aligned}$$

b. $\frac{x^{-2}y^3z^{-1}}{(xy^{-4})^{-2}z^3}$

$$\begin{aligned}
\frac{x^{-2}y^3z^{-1}}{(xy^{-4})^{-2}z^3} &= \frac{x^{-2}y^3z^{-1}}{x^{-2}(y^{-4})^{-2}z^3} \\
&= \frac{x^{-2}y^3z^{-1}}{x^{-2}y^8z^3} \\
&= \frac{x^2y^3}{x^2y^8z^3z} \\
&= \frac{x^2y^3}{x^2y^8z^4} \\
&= \frac{1}{y^5z^4}
\end{aligned}$$

c. $\sqrt{25x^3y^2}\,\sqrt[3]{8x^2y^7}$

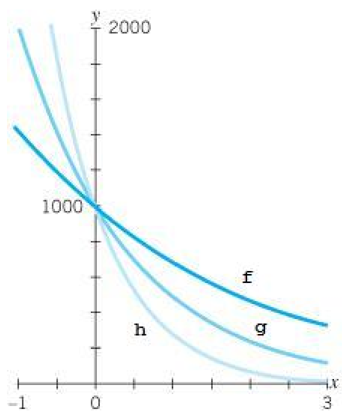
$$\begin{aligned}\sqrt{25x^3y^2}\,\sqrt[3]{8x^2y^7} &= \sqrt{25}\sqrt{x^3}\sqrt{y^2}\,\sqrt[3]{8}\,\sqrt[3]{x^2}\,\sqrt[3]{y^7}\\ &= \sqrt{25}\,\sqrt[3]{8}\sqrt{x^3}\,\sqrt[3]{x^2}\sqrt{y^2}\,\sqrt[3]{y^7}\\ &= 5\cdot 2\cdot (x^3)^{\frac{1}{2}}(x^2)^{\frac{1}{3}}(y^2)^{\frac{1}{2}}(y^7)^{\frac{1}{3}}\\ &= 10x^{\frac{3}{2}}x^{\frac{2}{3}}yy^{\frac{7}{3}}\\ &= 10x^{\frac{13}{6}}y^{\frac{10}{3}}\end{aligned}$$

[15] 4. Match each graph with the appropriate function.
Recall the worksheet we did in class about exponential functions. The important things were

- The larger the growth factor (when $a > 1$), the steeper the graph of the growth function.
- The smaller the decay factor (when $0 < a < 1$), the steeper the graph of the decay function.

You might combine these two facts into a single statement: The farther a is from 1, the steeper the graph of the function.

$1000 \cdot 0.3^x$, Graph h
$1000 \cdot 0.5^x$, Graph g
$1000 \cdot 0.7^x$, Graph f



[30] 5. The growth of a population of 100 bacteria is represented by the function $f(T) = 100 \cdot 3^T$, where T is measured in 5-minute intervals.

a. Use $f(T)$ to determine the number of bacteria present after 20 minutes.

20 minutes is the same as 4 5-minute intervals, so we plug in 4 for T.

$$\begin{aligned} f(4) &= 100 \cdot 3^4\\ &= 8100\end{aligned}$$

b. Set up but do not solve an equation which can be used to determine the amount of time required for the population to grow to 1000 bacteria.

The output of the function (the size of the population) is given, but we don't know the input. The equation we would need to solve, then, is

$$1000 = 100 \cdot 3^T$$

c. Use $f(T)$ to construct a new model $f(t)$, where t is measured in 1-minute intervals.

Think about the values of T and t when, say, 5 minutes have passed. $T = 1$, since it measures 5-minute intervals. $t = 5$, since it measures 1-minute intervals (i.e. the number of minutes). We see, then, that $T = \frac{t}{5}$. We plug this new representation for T into our old function to get:

$$\begin{aligned} f(t) &= 100 \cdot 3^{\frac{t}{5}} \\ &= 100 \cdot (3^{\frac{1}{5}})^t \\ &\approx 100 \cdot 1.25^t \end{aligned}$$

[30] 6. At the close of the 2000 fiscal year, a company netted $4 million in profit. Over the course of the following year, profits decreased by 7%. Assuming profits continue to decrease at this rate, construct an exponential function $f(t)$ that models the decay, where t denotes the number of years since 2000 and $f(t)$ denotes millions of dollars profit.

We should always start by seeing if the initial value, C, is given. Since the function starts measuring from the year 2000, the initial value will be the profit in that year. The problem tells us that this is 4 million dollars, so we want $C = 4$ (not 4,000,000, since the function denotes millions of dollars of profit).

When the change is given as a percentage, it is called a growth/decay rate instead of a growth/decay factor. The first thing we have to do is divide the percentage by 100 to get its decimal representation. The result is still a growth/decay rate, however, and so we can't use it in our function. We need to use the appropriate formula to change from a rate to a factor.

- growth factor = 1 + growth rate
- decay factor = 1 - decay rate

The word "decreased" in the problem tells us that we are talking about decay, so we get the decay factor

$$\begin{aligned} a &= 1 - .07 \\ &= .97 \end{aligned}$$

We have C and a, so our function is

$$f(t) = 4 \cdot .97^t$$

[30] 7. The population of a small town was 2,500 in 1950. By 2000, the population had grown to 40,000. Construct an exponential function $f(T)$ to model the population growth, where T denotes the number of 10-year periods since 1950.

The initial value is 2,500, since this is the population in 1950 (the first year the function cares about). If we go ahead and update our function, we have

$$f(T) = 2500 \cdot a^T$$

This is an input-output-unknown, so we can plug in another point to solve for a. The tricky part is deciding what value of T goes with the population 40,000. It takes 50 years to get from 1950 to 2000, but T measures the number of 10-year periods, so we want $T = 5$, not $T = 50$.

$$\begin{aligned} 40000 &= 2500 \cdot a^5 \\ 16 &= a^5 \\ 16^{\frac{1}{5}} &= (a^5)^{\frac{1}{5}} \\ 1.74 &\approx a \end{aligned}$$

We have C and a, so our function is

$$f(T) = 2500 \cdot 1.74^T$$

[30] 8. An unknown quantity of a chemical is present at the beginning of a reaction. Three minutes after the reaction begins, 40 mg of the chemical remain. Seven minutes after the reaction begins, 10 mg of the chemical are remain. Construct an exponential function $f(t)$ to model the decay where t denotes the number of minutes since the beginning of the reaction.

As always, we start by checking whether the initial value is given. In this problem, it is not, since we don't know how many miligrams of the chemical are present at the start of the reaction. Instead, we have to try to solve for a. To do this, we have to think about how exponential functions really work (which is why I keep assigning this kind of problem).

Since the function is supposed to model the decay of the chemical, let's think about the chemical first. We were given some unknown amount of the chemical to start with. We put it in a beaker and heat it up and, after 3 minutes, there is only 40 mg left. After another 4 minutes (7 minutes total), there is only 10 mg left. Now, think about our function. We're supposed to be able to get from one minute to the next simply by multiplying by some particular value of a. We don't know a, but we do know that multiplying 40 by a 4 times will give us 10. Why do we know this? At one point in the reaction, we had 40 mg. Four minutes later (i.e. multiplying by a four times), we ended up with 10 mg. If we make a little equation out of this, we can solve for a.

$$\begin{aligned} 40a^4 &= 10 \\ a^4 &= \frac{1}{4} \\ (a^4)^{\frac{1}{4}} &= \left(\frac{1}{4}\right)^{\frac{1}{4}} \\ a &\approx .71 \end{aligned}$$

We have a now, so we can update our function.

$$f(t) = C \cdot .71^t$$

We're in the input-output-unknown situation again, so we can plug in any point and solve for C. I'll use the point (3,40).

$$\begin{aligned} 40 &= C \cdot .71^3 \\ 40 &\approx C \cdot .36 \\ 111 &\approx C \end{aligned}$$

We have C and a, so our function is

$$f(t) = 111 \cdot .36^t$$

This problem involved a considerable amount of rounding, so your answer may be slightly different.