University of South Carolina
Math 111i: College Algebra
Instructor: Austin Mohr
Section 1
Spring 2009

Test 1
(200 points total)

[20] 1. Find the equation of the line containing the points (-2,4) and (5,-10).

All lines can be written as $y = b + mx$. Since the value of b is not given in the problem, we must determine m first using the slope formula.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-10 - 4}{5 - (-2)} \\ &= \frac{-14}{7} \\ &= -2 \end{aligned}$$

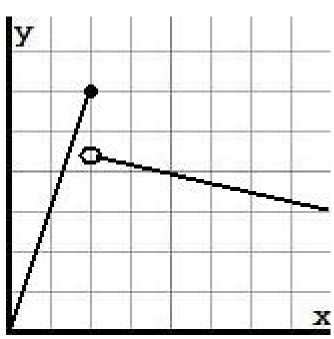
So, $y = b - 2x$. Now, we have input, output, and unknown, so plug in a valid point and solve for b.

$$\begin{aligned} 4 &= b - 2 * (-2) \\ 4 &= b + 4 \\ 0 &= b \end{aligned}$$

The entire equation is $y = 0 - 2x$, or just $y = -2x$.

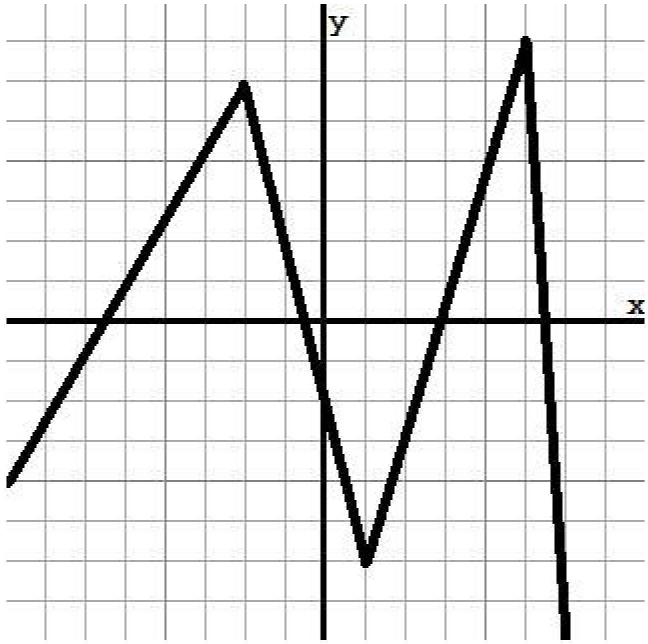
[20] 2. Graph the piecewise function

$$f(x) = \begin{cases} 3x & \text{if } x \le 2 \\ 5 - \frac{1}{4}x & \text{if } x > 2 \end{cases}$$



Remember, it is easiest to graph the entire line first (ignoring the instructions about x), then erase the part of the line you don't want.

[20] 3. The graph below represents a function $f(x)$.



a. On what interval(s) is $f(x)$ increasing?
$(-\infty, -2]$ and $[1, 5]$

b. On what interval(s) is $f(x)$ decreasing?
$[-2, 1]$ and $[5, \infty)$

c. Does $f(x)$ have a global maximum? If so, where does it occur and what is its value?
The max occurs at $x = 5$ with value $f(5) = 7$.

d. Does $f(x)$ have a global minimum? If so, where does it occur and what is its value?
There is no global minimum.

[21] 4. Find the domain of each of the following.

a. $f(x) = \frac{4x+1}{2x-5}$
We want the denominator not equal to 0. That is

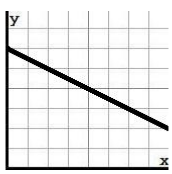
$$\begin{aligned} 2x - 5 &\neq 0 \\ 2x &\neq 5 \\ x &\neq \frac{5}{2} \end{aligned}$$

b. $f(x) = x^5 + 2x^3 - 1$
There is no division and no square roots, so the domain is all reals.

c. $f(x) = \sqrt{1-3x}$
We want the inside of the square root greater than or equal to 0. That is

$$\begin{aligned} 1 - 3x &\geq 0 \\ 1 &\geq 3x \\ \frac{1}{3} &\geq x \end{aligned}$$

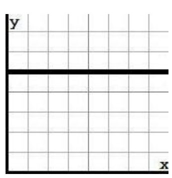
[21] 5. Determine the equation of the line in each graph below.



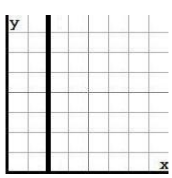
The y-intercept is $(0, 6)$, so $b = 6$. To find the slope, pick any two points on the line and use the slope formula. I'll pick $(0, 6)$ and $(2, 5)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5-6}{2-0} \\ &= \frac{-1}{2} \end{aligned}$$

So, $y = 6 - \frac{1}{2}x$.



The output is always 5, so the equation is $y = 5$.



This line does not specify a function, so we cannot use the normal $y = b + mx$ form. The only thing we can say about it is that x is always 2, so the best equation we can come up with is $x = 2$.

[20] 6. A presription coverage plan charges a base fee of \$500 annually and then \$20 per presription.

a. Represent the annual prescription cost as a function of the number of prescriptions filled.

First notice that this really is a linear function, since you figure out the cost by repeated addition (as opposed to, say, repeated multiplication). You pay \$500 if you buy 0 prescriptions, so $b = 500$. For each perscription you purchase, you pay another \$20, so $m = 20$. This makes our linear function $f(x) = 500+20x$.

b. What is a reasonable domain for the function? What is a reasonable range for the function?

It doesn't make sense to purchase a negative number of prescriptions, but any positive number makes sense. A reasonable domain would be $x \geq 0$.

[22] 7. You make monthly deposits into a savings account, depositing the same amount each month. You've lost all your records except for two receipts. In the twenty-third month, the account's balance was \$4375. In the forty-fifth month, the account's balance was \$7125. Write an equation $A(x)$ that models the value of the account as a function of the month. Do not assume that the account was empty when it first opened.

Like in problem 6, this really is a linear function because you determine the balance of the account by repeated addition. In the twenty-third month, the account balance was \$4375. We can treat this is the point $(23, 4375)$ on our graph. Similarly, the point $(45, 7125)$ is also on the graph. We know how to find the equation of a line through two points (we did it in problem 1), so we can do it again here.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7125 - 4375}{45 - 23} \\ &= \frac{2750}{22} \\ &= 125 \end{aligned}$$

So our function looks like $A(x) = b + 125x$. We are in the input-output-unknown situation, so we can plug in a point and solve for b.

$$\begin{aligned} 4375 &= b + 125 \cdot 23 \\ 4375 &= b + 2875 \\ 1500 &= b \end{aligned}$$

So our function is $A(x) = 1500 + 125x$.

[28] 8. The temperature on a particular evening dropped at a constant rate from 6:00pm to 7:00pm. The situation can be modeled by the function $F(t) = 75-2t$, where $F(t)$ is the temperature in degrees Fahrenheit and t is the number of 10-minute intervals since 6:00pm (so $t = 1$ is 6:10pm, $t = 2$ is 6:20pm, etc.).

a. What does the number 75 represent in the context of this problem?

75 is the y-intercept, which can also be interpreted as the initial value. This means that the temperature at 6:00pm was 75 degrees.

b. What does the number -2 represent in the context of this problem?

-2 is the slope, which is the rate of change. This means that the temperature was decreasing at a rate of 2 degrees per 10-minute interval.

c. What was the temperature at 6:40pm?

At 6:40pm, 4 10-minute intervals have passed, so $t = 4$.

$$\begin{aligned} F(4) &= 75 - 2 \cdot 4 \\ &= 75 - 8 \\ &= 67 \end{aligned}$$

d. When was the temperature 72 degrees Fahrenheit?
We need to solve

$$72 = 75 - 2 \cdot t$$
$$-3 = -2 \cdot t$$
$$1.5 = t$$

If 1.5 10-minute intervals have passed, then the current time is 6:15pm.

[28] 9. Consider the following table.

Year	**Population** (millions)	**Rate of Change**
1900	5	—-
1910	9	0.4
1920	7	
1930	13	0.6
1940	17	0.4
1950	16	-0.1
1960	18	0.2

Note: The Rate of Change column shows the average rate of change over the previous 10 years.

a. Compute the value for the Rate of Change column in 1920.
The rate of change is just the slope, so

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7 - 9}{1920 - 1910} \\ &= \frac{-2}{10} \\ &= -.2 \end{aligned}$$

b. Estimate the population in 1933.
From the chart, we see that the population in 1930 was 13. Each year between 1930 and 1940, the population increased by .4 on average. So, the population in 1933 is $13 + .4 + .4 + .4 = 14.2$ (or $13 + .4 \cdot 3 = 14.2$). Notice that we picked the rate of change in the 1940 row, since these rates of change reflect the *previous* 10 years.

c. During which 10-year period was the growth the fasest?
The biggest rate of change is in the 1930 row, which represents the growth from 1920 to 1930.

d. Without graphing, determine if this is a linear function. Explain your answer.
This cannot be a linear function because the rates of change are not constant throughout the entire time period.