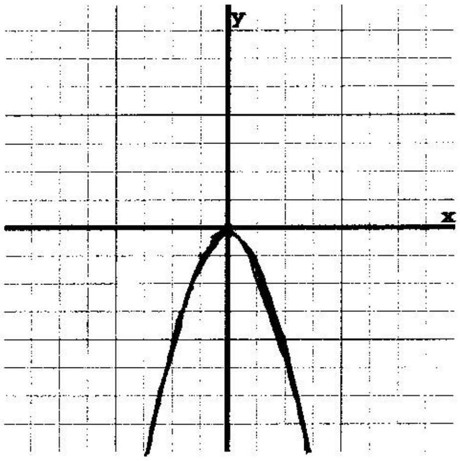
<u>**Quiz 9**</u>

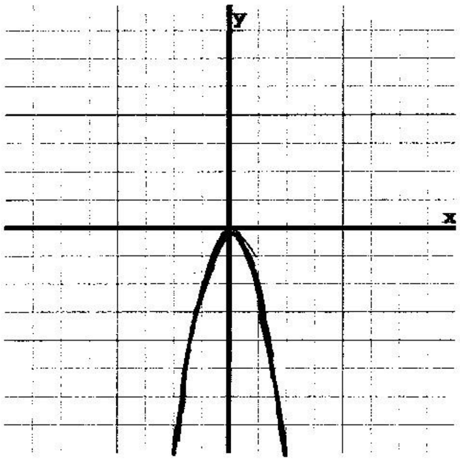
1. <u>Sketch</u> the graph of $f(x) = -2(x-1)^2 + 3$ by performing the translations one at a time. At each step, write the function of the graph you are sketching.

The order in which you perform the translations is irrelevant, so your steps may differ slightly from mine. The order I chose is

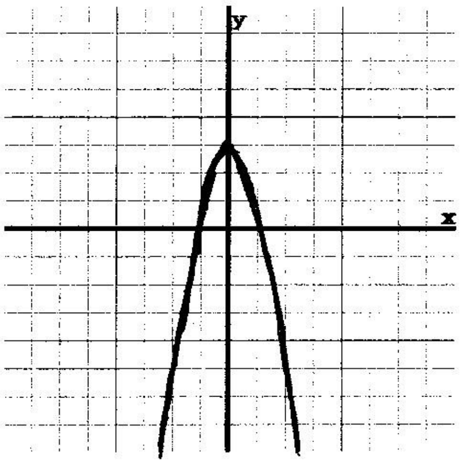
- vertical flip
- vertical stretch by a factor of 2
- vertical shift 3 units up
- horizontal shift 1 unit right



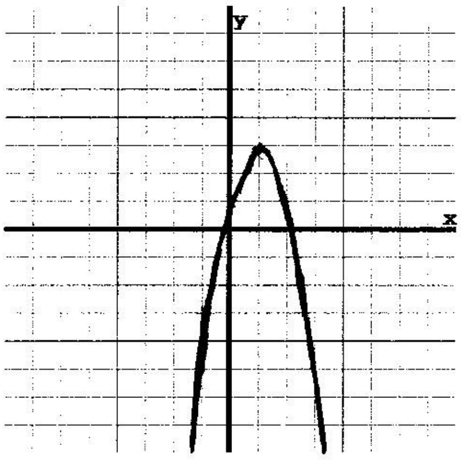

$-x^2$




$-2x^2$




$-2x^2 + 3$




$-2(x-1)^2 + 3$

2. Convert $f(x) = -2x^2 - 8x + 5$ to vertex form (i.e. $a(x-h)^2 + k$ form).

First we find the vertex using the fact that the x-coordinate is $\frac{-b}{2a}$

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{--8}{2\cdot(-2)} \\ &= \frac{8}{-4} \\ &= -2 \end{aligned}$$

The corresponding y-coordinate must be

$$\begin{aligned} f(-2) &= -2(-2)^2 - 8(-2) + 5 \\ &= -8 + 16 + 5 \\ &= 13 \end{aligned}$$

We now have the values for $h$ and $k$, so our function looks like

$$f(x) = a(x+2)^2 + 13$$

We're in the input-output-unknown situation, so we can plug in any point and solve for $a$. From the original function, we see that $(0, 5)$ lies on the graph (just plug in 0 for $x$), so we'll use that

$$\begin{aligned} 5 &= a(0+2) + 13 \\ 5 &= 2a + 13 \\ -8 &= 2a \\ -4 &= a \end{aligned}$$

So, the function is

$$f(x) = -4(x+2)^2 + 13$$

3. The function $h(t) = -t^2 + 20t + 25$ represents the height (in feet) of a particular object $t$ seconds after it is thrown.

a. What is the initial height of the object (i.e. the height of the object before it is thrown)?
The initial height occurs when $t = 0$, so we just plug 0 into $h$

$$\begin{aligned} h(0) &= -0^2 + 20\cdot 0 + 25 \\ &= 25 \end{aligned}$$

So, the object has an initial height of 25 feet.

b. When does the object reach maximum height? What is this height?
The maximum height occurs at the vertex, so we use the same formula as in part (b).

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-20}{2\cdot(-1)} \\ &= 10 \end{aligned}$$

So, the maximum height occurs 10 seconds after release. The maximum height is just the y-coordinate of the vertex, so we get

$$\begin{aligned} f(10) &= -10^2 + 20 \cdot 10 + 25 \\ &= -100 + 200 + 25 \\ &= 125 \end{aligned}$$

So, the maximum height is 125 feet.

c. When does the object hit the ground?
Another way of saying that the object is on the ground is to say $h(t) = 0$ (since the output of the function *is* the height). So, we need to solve

$$0 = -t^2 + 20t + 25$$

This is too hard to factor, so we should use the quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-20 \pm \sqrt{20^2 - 4 \cdot (-1) \cdot 25}}{2 \cdot (-1)} \\ &= \frac{-20 \pm \sqrt{400 + 100}}{-2} \\ &= \frac{-20 \pm \sqrt{500}}{-2} \\ &\approx -1.18 \text{ and } 21.18 \end{aligned}$$

The negative answer is meaningless in the context of the problem, so we conclude that the object hits the ground 21.18 seconds after release.