Quiz 8

1. For each part, pick the function that exhibits the specified behavior.

$f(x) = 10x$, $g(x) = 10^x$, $h(x) = log(x)$

a. If the input is increased by 1, the output is multiplied by 10. $g(x)$

b. If the input is multiplied by 10, the output is increased by 1. $h(x)$

c. If the input is increased by 1, the output is increased by 10. $f(x)$

Remember that linear functions are repeated addition, exponentional functions are repeated multiplication, and logarithmic functions are stubborn - you have to multiply the input just to get the output to increase.

2. Recall that the pH function is defined as

$$pH(H^+) = -\log(H^+)$$

where H^+ is the hydrogen ion concentration of a substance in $\frac{mol}{L}$. Recall also that the lower a substances pH, the more acidic it is.

An ulcer patient has been told to avoid acidic foods. If he drinks coffee, which has a pH of 5.0, it bothers him, but he can tolerate milk, which has a pH of 6.9.

a. Find the hydrogen ion concentration of coffee and the hydrogen ion concentration of milk.

For coffee

$$\begin{aligned} 5.0 &= -\log(H^+) \\ -5.0 &= \log(H^+) \\ 10^{-5.0} &= 10^{log(H^+)} \\ 10^{-5.0} &= H^+ \end{aligned}$$

For milk

$$\begin{aligned} 6.9 &= -\log(H^+) \\ -6.9 &= \log(H^+) \\ 10^{-6.9} &= 10^{log(H^+)} \\ 10^{-6.9} &= H^+ \end{aligned}$$

b. The patient is told that he can tolerate foods with a pH of 6.0, but nothing more acidic than that. Can he tolerate a half coffee-half milk mixture? (Hint: The hydrogen ion concentration of the mixture will be the average of the two concentrations from part (a).)

The hydrogen ion concentration of the mixture is

$$\frac{10^{-5.0} + 10^{-6.9}}{2} \approx 5.06 \cdot 10^{-6}$$

So, the pH of the mixture is

$$\begin{aligned} pH(5.06 \cdot 10^{-6}) &= -log(5.06 \cdot 10^{-6}) \\ &\approx 5.30 \end{aligned}$$

The pH of the mixture is lower than 6.0, so it is too acidic (since lower pH means more acidic).

3. Let $f(x) = x^2 - 9$.

a. What is the y-intercept?

The y-intercept is the point on the graph where $x = 0$, so we compute

$$\begin{aligned} f(0) &= 0^2 - 9 \\ &= -9 \end{aligned}$$

So, the y-intercept is the point $(0, -9)$ (not just the number -9).

b. What are the roots? (Hint: You don't need any special techniques or formulas for this.)

The roots are the points on the graph where $f(x) = 0$, so we solve

$$\begin{aligned} x^2 - 9 &= 0 \\ x^2 &= 9 \\ x &= 3 \text{ or } -3 \end{aligned}$$

So, the roots are the points $(3, 0)$ and $(-3, 0)$ (not just the numbers 3 and -3).

c. What is the vertex? (Hint: The x-coordinate of the vertex is halfway between the roots. You can then use the x-coordinate to find the y-coordinate.)

Halfway between 3 and -3 is 0, so the x-coordinate of the vertex is 0. To find the y-coordinate, we plug 0 into the function to get $f(0) = -9$. So, the vertex is the point $(0, -9)$. Notice this is the same as the y-intercept, which can happen sometimes.

d. What is the equation of the axis of symmetry?

The axis of symmetry is the vertical line passing through the vertex, so its equation is $x = 0$.