Quiz 7

1. Suppose you invest P_0 dollars at 9% interest.

Recall the relevant formulas. If you have initial value C and (nominal) interest rate r, then the value of your investment after t years is given by one of the following:

- Interest compounded n times: $f(t) = C(1 + \frac{r}{n})^{nt}$
- Interest compounded continuously: $f(t) = Ce^{rt}$

a. Write a function to model the growth of the investment if it is compounded annually.

$$\begin{aligned} f(t) &= P_0(1 + \frac{0.09}{1})^{1t} \\ &= P_0 \cdot 1.09^t \end{aligned}$$

b. Write a function to model the growth of the investment if it is compounded every four months.

$$\begin{aligned} f(t) &= P_0(1 + \frac{0.09}{3})^{3t} \\ &= P_0 \cdot 1.03^{3t} \\ &= P_0 \cdot (1.03^3)^t \\ &= P_0 \cdot 1.093^t \end{aligned}$$

For this one, you could have stopped at even the first line. The extra work I've done was to make sure that the function is in Ca^t form. If there are questions about the growth factor or growth rate of this function, then you must get the function into this form in order to answer them.

c. Write a function to model the growth of the investment if it is compounded continuously.

$$f(t) = P_0e^{0.09t}$$

d. Convert your answer to part (c) to Ca^t form. The function is currently in Ce^{kt} form and we want to rewrite it as Ca^t. So, it must be that $a = e^k$ (this is how you start any of these conversion problems).

$$\begin{aligned} a &= e^k \\ &= e^{0.09} \\ &\approx 1.094 \end{aligned}$$

So, we can write the function as $f(t) = P_0 \cdot 1.094^t$.

2. The decay of 300mg of strontium-90 can be modeled by the function $f(t) = 300 \cdot 0.976^t$, where t is measured in years.

a. Identify the initial value, decay factor, decay rate, and continuous decay rate. (Hint: You may have to convert to another form to determine some of these.)

The function is currently in Ca^t form, so we can figure out the first three items.

- initial value: 300
- decay factor: 0.976
- decay rate: 0.024 (using decay factor = 1 - decay rate)

The continuous decay rate is specific to the Ce^{kt} form, so we will have to convert. We start in the same way as in problem 1.

$$\begin{aligned} a &= e^k \\ 0.976 &= e^k \\ ln(0.976) &= ln(e^k) \\ -0.024 &= k \end{aligned}$$

So, the continuous decay right -0.024.

b. How much strontium-90 remains after 5 years?

For the rest of these problems, we can use either form. I will just use the original Ca^t form.

$$\begin{aligned} f(5) &= 300 \cdot 0.976^5 \\ &= 300 \cdot 0.886 \\ &= 265.8 \end{aligned}$$

c. How long will it take for the sample to decay to 10mg?

$$\begin{aligned} 10 &= 300 \cdot 0.976^t \\ 0.033 &= 0.976^t \\ ln(0.033) &= ln(0.976^t) \\ ln(0.033) &= t \cdot ln(0.976) \\ -3.411 &= t(-0.024) \\ 142.125 &= t \end{aligned}$$

d. What is the half-life of strontium-90?

$$\begin{aligned} 150 &= 300 \cdot 0.976^t \\ 0.5 &= 0.976^t \\ ln(0.5) &= ln(0.976^t) \\ ln(0.5) &= t \cdot ln(0.976) \\ -0.693 &= t(-0.024) \\ 28.875 &= t \end{aligned}$$

e. Suppose an unknown amount of strontium-90 is released into a room. Fifty years later, scientists enter the room and determine that 250mg of the chemical still remain. How much was originally released into the room?

$$\begin{aligned} 250 &= C \cdot 0.976^{50} \\ 250 &= C \cdot 0.297 \\ 841.751 &= C \end{aligned}$$