Quiz 6

1. Expand.

$$log\left(\frac{x\sqrt[3]{x+2}}{(y-1)^3}\right)$$

$$log(x\sqrt[3]{x+2}) - log((y-1)^3) \qquad \text{(since } log\left(\frac{A}{B}\right) = log(A) - log(B)\text{)}$$
$$log(x) + log(\sqrt[3]{x+2}) - log((y-1)^3) \qquad \text{(since } log(AB) = log(A) + log(B)\text{)}$$
$$log(x) + \frac{1}{3}log(x+2) - 3log(y-1) \qquad \text{(since } log(A^p) = plog(A)\text{)}$$

2. Solve for x.

$log(x^5) - log(x^3) + 2log(x) = 8$

There are two ways to approach this. One is to contract.

$$log(x^5) - log(x^3) + 2log(x) = 8$$
$$log\left(\frac{x^5}{x^3}\right) + 2log(x) = 8 \qquad \text{(since } log\left(\frac{A}{B}\right) = log(A) - log(B)\text{)}$$
$$log(x^2) + 2log(x) = 8$$
$$log(x^2) + log(x^2) = 8 \qquad \text{(since } log(A^p) = plog(A)\text{)}$$
$$log(x^2x^2) = 8 \qquad \text{(since } log(AB) = log(A) + log(B)\text{)}$$
$$log(x^4) = 8$$
$$10^{log(x^4)} = 10^8 \qquad \text{(since } 10^{LHS} = 10^{RHS}\text{)}$$
$$x^4 = 10^8 \qquad \text{(since } 10^{logx} = x\text{)}$$
$$(x^4)^{\frac{1}{4}} = (10^8)^{\frac{1}{4}}$$
$$x = 10^2$$
$$x = 100$$

Alternatively, you could expand.

$$log(x^5) - log(x^3) + 2log(x) = 8$$
$$5log(x) - 3log(x) + 2log(x) = 8$$
$$4log(x) = 8 \qquad \text{(5 apples - 3 apples + 2 apples = 4 apples)}$$
$$log(x) = 2$$
$$10^{log(x)} = 10^2 \qquad \text{(since } 10^{LHS} = 10^{RHS}\text{)}$$
$$x = 100 \qquad \text{(since } 10^{logx} = x\text{)}$$

3. The decay of 50 mg of a chemical is modeled by $f(t) = 50 \cdot .9^t$, where t is measured in years.
   a. How much time is required for the sample to decay to 5 mg?

We know the function's output (number of miligrams), but not the input (time). So, we need to solve

$$\begin{aligned} 5 &= 50 \cdot .9^t \\ .1 &= .9^t \\ log(.1) &= log(.9^t) \\ log(.1) &= tlog(.9) && \text{(since } log(A^p) = plog(A)\text{)} \\ \frac{log(.1)}{log(.9)} &= t \\ 21.85 &\approx t \end{aligned}$$

So, about 21.85 hours are required for the sample to decay to 5 mg.
b. What is the half-life of this chemical?

Recall a couple of definitions:

- doubling time: the amount of time required for the initial value to grow to twice its size
- half-life: the amount of time required for the initial value to decay to half its size

Since we our initial value was 50 mg, we want to know when the sample will decay to 25 mg. So, we need to solve

$$\begin{aligned} 25 &= 50 \cdot .9^t \\ .5 &= .9^t \\ log(.5) &= log(.9^t) \\ log(.5) &= tlog(.9) && \text{(since } log(A^p) = plog(A)\text{)} \\ \frac{log(.5)}{log(.9)} &= t \\ 6.68 &\approx t \end{aligned}$$

So, the half-life of this chemical is about 6.68 hours.