Quiz 5

1. Consider the following exponential functions.

$f_1(x) = (\frac{1}{4})^x$, $f_2(x) = (\frac{1}{2})^x$, $f_3(x) = (\frac{3}{4})^x$, $f_4(x) = (\frac{5}{4})^x$, $f_5(x) = (\frac{3}{2})^x$, $f_6(x) = (\frac{7}{4})^x$

Which growth function has the steepest graph? f_6
Which growth function has the shallowest graph? f_4
Which decay function has the steepest graph? f_1
Which decay function has the shallowest graph? f_3

Growth functions require $a > 1$ and larger a's give faster growth (and so steeper graphs).
Decay functions require $0 < a < 1$ and smaller a's give faster decay (and so steeper graphs).

2. Chemical X decays by 60% every 5 days.
   a. Model the decay of a 50mg sample of Chemical X as a function $f(T)$, where T is 5-day periods.

Decay Factor = 1 - Decay Rate
Decay Factor = 1 - .6 = .4
Each step in T is exactly one "round" of decay, since it takes 5 days to experience the full decay factor and each step in T represents 5 days. So, we have

$$f(T) = 50 \cdot .4^T$$

   b. Use the function from part (a) to construct a new function $f(t)$, where t is 1-day periods.

When $T = 1$, $t = 5$. When $T = 2$, $t = 10$. So, we see that $T = \frac{t}{5}$. Plug this into $f(T)$ to get

$$\begin{aligned} f(t) &= 50 \cdot .4^{\frac{t}{5}} \\ &= 50 \cdot (.4^{\frac{1}{5}})^t \\ &\approx 50 \cdot .833^t \end{aligned}$$

3. Find an exponential function through the points (2, 100) and (12, 5).

We know that $f(x) = Ca^x$. We usually start by finding C, but that is impossible here, since we don't have the point $(0, C)$. So, we have to start by finding a. To do this, we have to stop and consider how exponential functions work.

If you know the output of an exponential function for, say, $t = 0$, how do you find the output for $t = 1$? You multiply by the growth factor a. If you have the output for $t = 1$, how do you get the output for $t = 2$? You multiply by the growth factor a. In fact, to move from any output value to any later output value, you just have to multiply by a the right number of times.

Let's look out our example now. When $t = 2$, the output is 100. When $t = 12$, the output is 5. So, we should be able to multiply 100 by a a certain number of times to get 5. In other words

$$100 \cdot a^? = 5$$

How many times do we need to apply a? We started at $t = 2$ and we need to get to $t = 12$, so we need 10 applications of a to get to where we need to be. So,

$$100 \cdot a^{10} = 5$$

Now we can solve for a.

$$
\begin{aligned}
100 \cdot a^{10} &= 5 \\
a^{10} &= .05 \\
a &= .05^{\frac{1}{10}} \\
a &\approx .741
\end{aligned}
$$

(Alternatively, you may use the cumbersome formula

$$a = (\tfrac{y_2}{y_1})^{\frac{1}{x_2 - x_1}}$$

but I wanted to teach you a more enlightening approach first.)

Now, we have

$$f(x) = C \cdot .741^x$$

We can find C by plugging in any point and solving.

$$
\begin{aligned}
&100 = C \cdot .741^2 \\
&100 \approx C \cdot .549 \\
&182 \approx C
\end{aligned}
$$

So, our function is

$$f(x) = 182 \cdot .741^x$$