Quiz 10

1. Consider the polynomial function $f(x) = -3x^3 - 3x^2 + 18x$.

a. What is the maximum number of turning points this function can have? What is the maximum number of roots this function can have?
A polynomial of degree n can have at most $n-1$ turning points and n roots. This particular polynomial is of degree 3, which means it can have at most 2 turning points and 3 roots.

b. Find the y-intercept.
As with any function, the y-intercept is the value at $x = 0$, which in this case is 0 (just plug 0 in for x). So the y-intercept is (0,0).

c. Find the roots. (Hint: Factor the polynomial. Be sure to look for common factors.)
The biggest common factor is $-3x$ (you don't have to take the negative sign, but it makes the rest easier), so the partially factored form is

$$-3x(x^2 + x - 6)$$

Now we need to factor the part in parentheses. Looking at the factors of -6, we see that the pair 3, -2 add up to 1. So, the fully factored form is

$$-3x(x + 3)(x - 2)$$

To find the roots, we want to solve

$$-3x(x + 3)(x - 2) = 0$$

To do this, we solve each factor separately.

$$\begin{aligned} -3x &= 0 \\ x &= 0 \end{aligned}$$

or

$$\begin{aligned} x + 3 &= 0 \\ x &= -3 \end{aligned}$$

or

$$\begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}$$

So, the roots are (0, 0) (which also happens to be the y-intercept), (-3, 0), and (2, 0).

d. Describe the global behavior.
When determining the global behavior, we only need to consider the leading term (the farther we get from the origin, the less the other terms contribute to the behavior). The leading term here is $-3x^3$. We can determine the behavior by just looking at any positive input and any negative input.

$$-3(1)^3 = -3$$

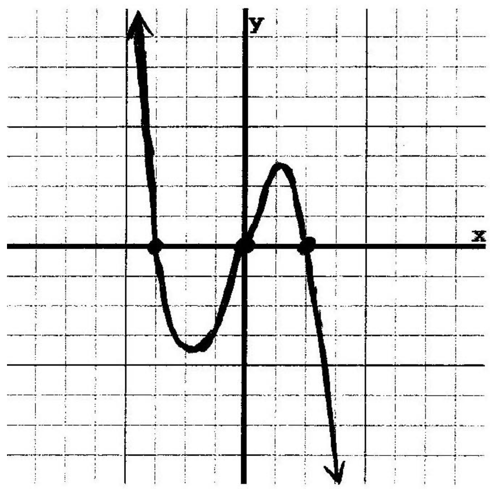
so if $x \to \infty$, then $y \to -\infty$.

$$-3(-1)^3 = 3$$

so if $x \to -\infty$, then $y \to \infty$.

e. Sketch the graph of this function. Make sure your sketch agrees with the information you obtained in the previous parts.

Start by plotting the y-intercept and all the roots we found. Next, use the end behavior to determine what happens to the left and right of these points. When $x \to \infty$, we found that $y \to -\infty$, so the function should decrease on the far right side of the graph. When $x \to -\infty$, we found that $y \to \infty$, so the function should increase on the far left side of the graph.



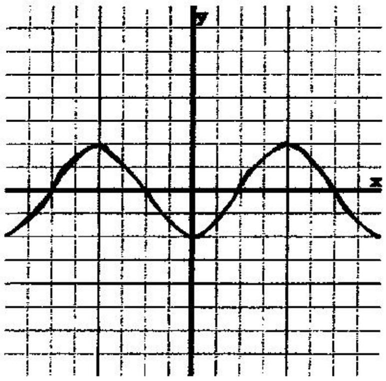
2. Construct a polynomial function with roots at $x = -2, 3$, and 4 passing through the point $(5, 21)$. Do not simplify your answer.

To get the specified roots, we write the factored form $(x+2)(x-3)(x-4)$. Notice there is one set of parentheses for each root that, if you plug in the root, you get 0. Notice that $a(x+2)(x-3)(x-4)$ also has these same roots for *any* value of a. To make sure the polynomial passes through the specified point, we plug in the point and solve for a.

$$\begin{aligned} f(x) &= a(x+2)(x-3)(x-4) \\ 21 &= a(5+2)(5-3)(5-4) \\ 21 &= 14a \\ \frac{21}{14} &= a \\ \frac{3}{2} &= a \end{aligned}$$

So, the polynomial we want is $f(x) = \frac{3}{2}(x+2)(x-3)(x-4)$.

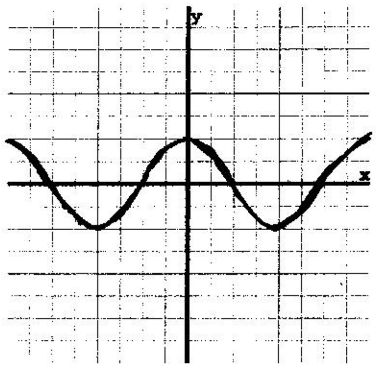
3. Suppose $f(x)$ has the following graph. (Assume that this pattern repeats forever.)



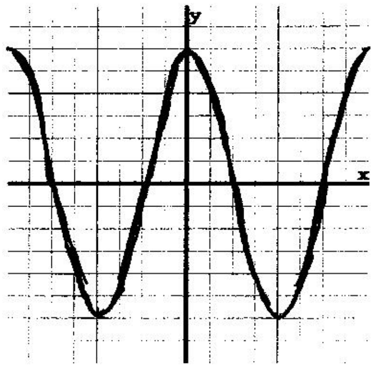
Sketch the graph of the following by applying transformations one at a time. At each step, write the equation (in terms of $f(x)$) of the function you are graphing.

a. $-3f(x+1)$

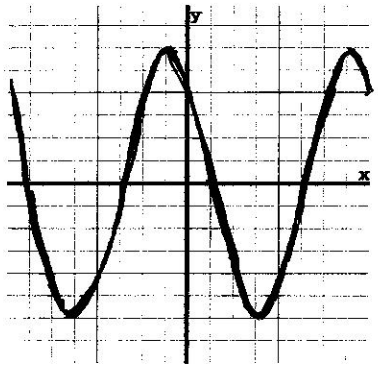
The transformations are a vertical flip, a vertical stretch by a factor of 3, and a horizontal shift to the left by 1 (remember the horizontal shift is the opposite of what it seems like it should be).



$-f(x)$



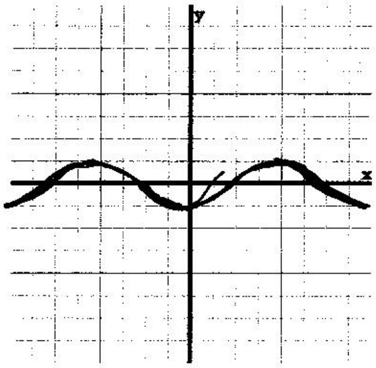
$-3f(x)$



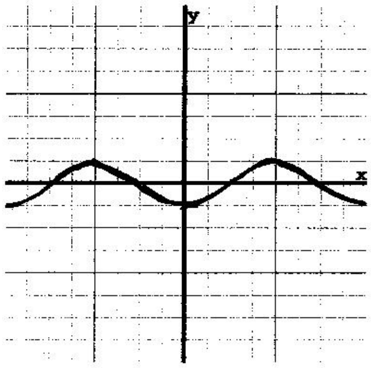
$-3f(x+1)$

b. $\frac{1}{2}f(-x)+3$

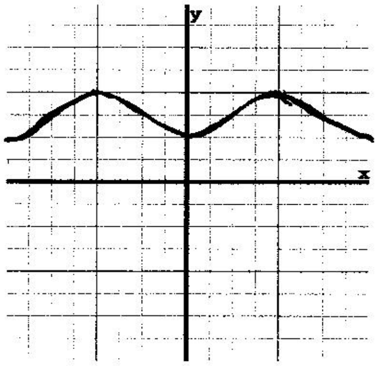
The transformations are a vertical compression by a factor of $\frac{1}{2}$, a horizontal flip (which, incidentally, does nothing since this particular graph is symmetric about the y-axis), and a vertical shift up 3.



$\frac{1}{2}f(x)$



$\frac{1}{2}f(-x)$



$\frac{1}{2}f(-x)+3$