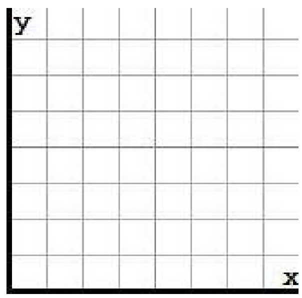
University of South Carolina
Math 111i: Intensive College Algebra
Instructor: Austin Mohr
Section 1
Spring 2009

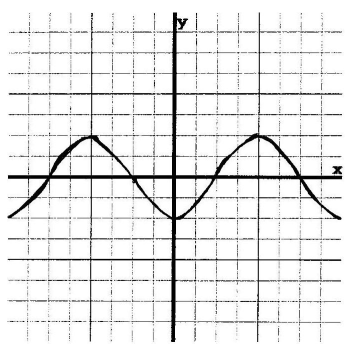
Final Exam
(400 points total)

[20] 1. Graph the following piecewise function. Note that you only have to graph the first quadrant (positive x and positive y).

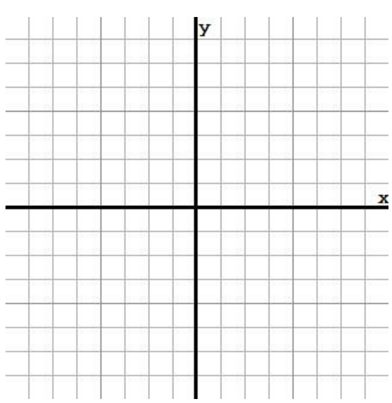
$$f(x) = \begin{cases} \frac{1}{2}x + 2 & \text{if } x \leq 5 \\ 2^{x-5} & \text{if } x > 5 \end{cases}$$

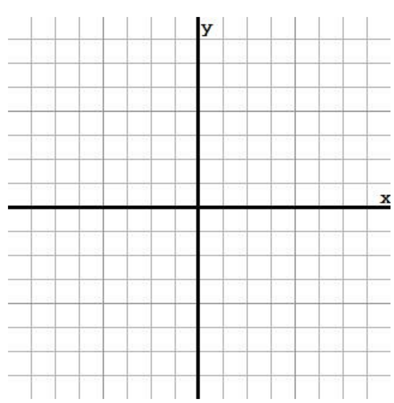


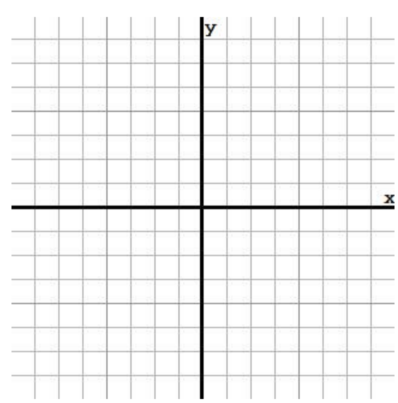
[20] 2. Suppose $f(x)$ has the following graph.



Sketch the graph of $-2f(x) - 4$ by applying transformations one at a time. At each step, write the equation (in terms of $f(x)$) of the function you are graphing.







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[35] 3. Consider the polynomial function $f(x) = x^4 + x^3 - 7x^2 - x + 6$.

a. What is the maximum number of turning points this function can have?

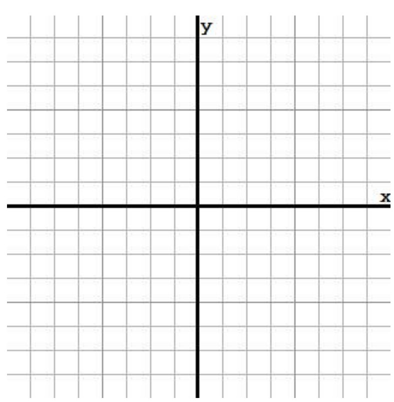
b. What is the maximum number of roots this function can have?

b. Find the y-intercept.

c. Use the fact that $f(x)$ factors as $(x^2 + 4x + 3)(x^2 - 3x + 2)$ to find the roots.

d. Describe the global behavior.

e. Sketch the graph of this function. Make sure your sketch agrees with the information you obtained in the previous parts.



[25] 4. Simplify. Your answer should not contain negative exponents or radicals.

$$\frac{\sqrt[3]{a^{-9}bc^{-4}}}{(ab^2)^{-2}c^{-3}}$$

[25] 5. Contract as much as possible.

$$(log(w) + \frac{1}{2}log(x-1))(4log(y) - log(z))$$

[25] 6. Convert 7 meters per hour to inches per minute. (Hint: 1 inch = 2.54 centimeters)

[30] 7. Give the domain for each of the following functions.

a. $f(x) = -2x^4 + x^2 + 5x - 8$

b. $f(x) = log(2x + 5)$

c. $f(x) = 20e^x$

d. $f(x) = \sqrt{-3x + 9}$

e. $f(x) = \frac{-2x-1}{x^2+5x-6}$

[35] 8. Solve for x.

a. $2log(x) + log\left(\frac{4}{x}\right) = 2$

b. $\frac{2x+30}{x-1} = 2x$

c. $5^{3x-1} = 6$

[40] 9. Give the function $f(x)$ that represents each of the following.

a. The polynomial with roots $x = -2$, $x = 1$, and $x = 3$ and passing through the point (2, 12)

b. A line through the points (1, 2) and (5, -6)

c. A line through the points (-2, 4) and (3, 4)

d. The graph of $y = 2^x$ flipped horizontally, stretched vertically by a factor of 3, and shifted up 4 units

e. The parabola with vertex (-2, 3) through the point (-5, 21)

f. The graph of $y = a \cdot ln(x)$ through the point (e^3, -9) (Hint: You need to solve for a.)

g. The exponential function passing through the points (3, 62.5) and (5, 390.625)

[40] 10. In each part, construct the requested function.

a. A petri dish contains 250 cells. This population triples every minute. Construct a function $f(t)$ that represents the number of bacertia present after t minutes.

b. The population of a certain town is currently 7500. The population is expected to increase by 10% each year. Construct a function $f(t)$ that represents the population after t years.

c. The ground is covered by five inches of snow. The snow melts at a rate of half an inch per hour. Construct a function $f(t)$ that represents the number of inches of snow still on the ground after t hours.

d. Suppose \$3000 is invested at 11% (nominal) annual interest. The interest is compounded continuously. Construct a function $f(t)$ that represents the value of the investment after t years.

e. The cost of membership at a health club is a \$100 one-time sign-up fee and \$35 per month. Construct a function $f(t)$ that represents the total cost of membership for t months.

f. A 40-milligram sample of a certain chemical begins to decay. It is known to have a half-life of one minute. Construct a function $f(t)$ that represents the number of milligrams remaining after t minutes.

[35] 11. A certain graduate student rudely drinks coffee while he lectures. The function $f(t) = 16 - \frac{1}{2}t$ represents the number of ounces of coffee remaining t minutes into the lecture.

a. How much coffee was initially in the cup?

b. How many ounces does he drink each minute?

c. How many ounces are in the cup after 15 minutes?

d. How much time is required to drink the entire cup?

e. What is a reasonable domain for this function?

f. What is a reasonable range for this function?

[35] 12. The function $f(T) = 35 \cdot 3^T$ represents the number of bacteria present after T 10-second intervals.

a. How many bacteria were initially present?

b. How does the size of the population change every 10-second interval?

c. How many 10-second intervals are required for the population to grow to 1000 bacteria? Round your answer to two decimal places.

d. How many bacteria are present after 2 minutes?

e. Convert $f(T)$ to Ce^{kT} form.

f. Construct a function $g(t)$ that represents the number of bacteria present after t 1-second intervals. Assume that the initial population size and growth rate are the same as in $f(T)$.

[35] 13. An object is thrown on the planet Pluto. The height (in feet) of the object t seconds after being thrown is given by

$$h(t) = 5 + 10t - t^2$$

a. What was the height of the object the moment it was thrown?

b. When did the object achieve maximum height? What was the maximum height?

c. When did the object hit the ground?