University of South Carolina
Math 111: College Algebra
Instructor: Austin Mohr
Section 8
Fall 2008

Final Exam
(400 points total)

[25] 1. Give the domain for each of the following functions.

a. $f(x) = \sqrt{-2x+3}$

b. $f(x) = \frac{3x+4}{x^2+7x+6}$

c. $f(x) = 3x^3 + 2x^2 + x + 2$

d. $f(x) = log(3x+1)$

e. $f(x) = 10e^{.04x}$

[35] 2. Give the function $f(x)$ that fits each description.

a. A line through the points (1, 3) and (3, -2)

b. A line through the points (1, 2) and (1, 3)

c. The parabola with vertex (-5, 6), opening downward, and compressed by a factor of $\frac{1}{3}$

d. The graph of $y = e^x$ flipped vertically, stretched vertically by a factor of 2, shifted right 3 units, and shifted up 1 unit

e. The exponential function $f(t) = Ca^t$ passing through the points (2, 22.5) and (4, 50.625)

f. The polynomial with roots $x = -1$, $x = 0$, and $x = 2$ and passing through the point (1, 8)

g. The graph of $y = a \cdot log(x)$ through the point (100, 6)

[30] 3. Consider the polynomial function $f(x) = -x^7 - 2x^6 + 8x^5 + x^3 + 2x^2 - 8x$.

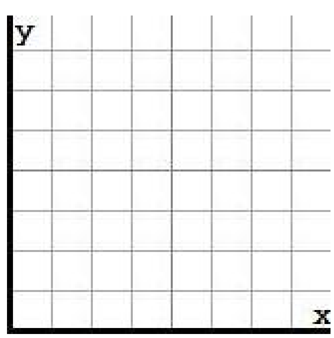
a. Describe the global behavior of $f(x)$. (How does $f(x)$ behave as $x \to \infty$? How does $f(x)$ behave as $x \to -\infty$?)

b. What is the maximum number of real roots this polynimal can have?

c. $f(x)$ factors as $-x(x^2 + 1)(x^2 - 1)(x^2 + 2x - 8)$. Use this fact to determine the real roots of $f(x)$.

[25] 4. Graph the piecewise function

$$f(x) = \begin{cases} (x-2)^2 + 1 & \text{if } x \leq 3 \\ 2^{x-3} & \text{if } x > 3 \end{cases}$$



[15] 5. Simplify. Your answer should not contain negative exponents or radicals.

$\frac{\sqrt{a^{-2}bc^{-3}}}{(ab^{-1})^{-2}c^2}$

[15] 6. Contract as much as possible.

$\frac{2log(w+1)-\frac{1}{2}log(x)}{3log(y)+log(z)}$

[20] 7. The average distance between the sun and the earth is 150 million kilometers.

a. Express this distance using scientific notation.

b. Recall that 1 inch = 2.54 centimeters. Convert 150 million kilometers to inches. Your answer should be in scientific notation.

[30] 8. Solve for x.

a. $2^{2x+1} = 5$

b. $log(x) - log(x+2) = 1$

c. $2x^2 - 2x - 15 = 3$

[40] 9. A particular savings account has a 5% (nominal) annual interest rate and is compounded continuously.

a. Construct a function $f(t)$ to model the growth of a \$5000 investment where t is measured in years.

b. What is the effective interest rate for this account? Give your answer as a percent rounded to two decimal places.

c. How many years are required for the investment to grow to \$7500?

d. Let the function $g(t)$ represent the growth of the same investment at the same interest rate, but invested 10 years later. Describe $g(t)$ as a transformation of $f(t)$. (For example, if you can get $g(t)$ by stretching $f(t)$ by a factor of 2, then $g(t) = 2f(t)$.) Specify the reasonable domain for $g(t)$.

[45] 10. A test conducted in 2004 reveals that a river is polluted with 285 parts per million (ppm) of a toxic substance. Local officials estimated they could reduce the pollution by 15 ppm each year.

a. Construct a function $f(t)$ to model the ppm of the pollutant in the river where t denotes the number of years since 2004.

b. Estimate the ppm of the pollutant in 2010.

c. The river will be safe for swimming when the concentration of the pollutant is reduced to 45 ppm. Estimate the year in which it will be safe to swim in the river.

[35] 11. Consider the following table.

Year	**Population** (millions)	**Rate of Change**
1900	22	—-
1905	26	
1910	33	1.4
1915	35	0.4
1920	32	-0.6
1925	31	-0.2
1930	36	1.0

Note: The Rate of Change column shows the average rate of change over the previous 5 years.

a. Compute the missing value in the Rate of Change column.

b. Estimate the population in 1927.

c. During which years was the population increasing? During which years was the population decreasing?

d. During which 5-year period was the population increasing at the fastest rate? During which 5-year period was the population decreasing at the fastest rate?

e. Does this data represent a linear function? Why or why not?

[45] 12. An object was thrown on the planet Pluto. The height (in feet) of the object t seconds after being thrown is given by

$$h(t) = 25 + 20t - t^2$$

a. What was the height of the object the moment it was thrown?

b. When did the object achieve maximum height? What was the maximum height?

c. When did the object hit the ground?

[40] 13. The population of a colony of bacteria has a doubling time of 15 seconds.

a. Construct a function $f(T)$ to model the growth of an arbitrary initial population P_0 where T is measured in 15-second intervals.

b. Construct a function $f(t)$ to model the growth of an arbitrary initial population P_0 where t is measured in 1-second intervals.

c. Suppose the initial size of the population is 30 bacteria. Determine the size of the population after 47 seconds.

d. Using the function $f(t)$ in part b, determine the continuous growth rate of the bacteria.

[20] **Extra Credit:** An infinite binary string is a sequence of 0's and 1's of infinite length. Let A denote the set of all infinite binary strings.

Proposition 1. *A is not countably infinite.*

Proof. Suppose, for the purpose of contradiction, that the set of all infinite binary strings is countably infinite. Then, there is a one-to-one, onto function $f : \mathbb{N} \to A$. Consider this function as a list:

n			**f(n)**			
0	1	0	0	0	1	...
1	0	1	1	0	1	...
2	1	1	0	0	0	...
3	0	1	1	0	1	...
4	0	0	1	0	0	...
⋮			⋮			

a. Show the first 5 digits of a binary string m which cannot be on the list.

b. Why is it impossible for m to be on the list? (Why can't it be the 10^{th} string on the list? Why can't it be the 356^{th} string on the list?)

We have shown that m, which is in the target set A, does not appear on the list above. In other words, f is not onto, which is a contradiction with our assumption that A is countably infinite. Therefore, A is not countably infinite.

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