Southern Illinois University
Math 107: Intermediate Algebra
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Sections 7 and 17
Fall 2007

Solving Word Problems

Our text lists six steps involved in solving word problems on page 79.

1. **Read** the problem, several times if necessary, until you understand what is given and what is to be found.
2. **Assign a variable** to represent the unknown value, using diagrams or tables as needed. Write down what the variable represents. Express any other unknown values in terms of the variable.
3. **Write an equation** using the variable expression(s).
4. **Solve** the equation.
5. **State the answer** to the problem. Does it seem reasonable?
6. **Check** the answer in the words of the original problem.

We will use this framework to carefully solve some examples from the text. Additionally, you may find it helpful to review the table on page 77 entitled "Translating from Words to Mathematical Expressions".

2.3.29
The John Hancock Center in Chicago has a rectangular base. The length of the base measures 65 ft. less than twice the width. The perimeter of this base is 860 ft. What are the dimensions of the base?
Read: We are given that the perimeter (the sum of the measures of all the sides of the rectangle) is 860 ft. We are also given a "fact" about the length of the rectangle: it is 65 ft. less than twice the rectangle's width. The problem asks us to find the dimensions (length and width in feet) of the rectangle.
Assign a variable: We have two quantities we need to find: length and width. Let's call them L and W, respectively.
Write an equation: One equation may not always be enough. It's best to write down everything you can, even if you don't end up using it. The following formulas will likely be important.

- Perimeter $= W + W + L + L = 2W + 2L = 860$
- $L = 2W - 65$

Solve the equation: We have two equations and two unknown values (L and W). (This is typically referred to as a system of equations.) To solve this, we make the observation that, even though we don't know *precisely* what L is, we do know that it is equal to $2W - 65$ (whatever W is). So, let's substitute this "value" for L into our first equation and solve for W.

$$\begin{aligned}
860 &= 2W + 2L && \text{(original formula)}\\
860 &= 2W + 2(2W - 65) && \text{(substitution)}\\
860 &= 2W + 4W - 130 && \text{(distributive property)}\\
860 &= 6W - 130 && \text{(combine like terms)}\\
860 + 130 &= 6W - 130 + 130 && \text{(cancel 130)}\\
990 &= 6W\\
\tfrac{990}{6} &= \tfrac{6W}{6} && \text{(isolate } W\text{)}\\
165 &= W
\end{aligned}$$

We're not quite finised. We have a value for W, but we also need one for L. From the problem, we know that $L = 2W - 65$. So, we plug in our value for W and find that $L = 2(165) - 65 = 265$.
State the answer: The solution $W = 165$ means "the width of the base is 165 ft.". Similarly, $L = 265$ means "the length of the base is 265 ft.". Both answers seem reasonable enough. An unreasonable answer would be a negative number or a number larger than the entire perimeter.
Check: With values for length and width, we can easily compute the perimeter of the base to check our answers.

$$\begin{aligned}
2W + 2L &= 2(165) + 2(265)\\
&= 330 + 530\\
&= 860
\end{aligned}$$

This agrees with the information provided in the problem, so we know our answer is right.

2.3.45
Julie Gasway invested some money at 4.5% simple interest and \$1000 less than twice this amount at 3%. Her total annual income from the interest was \$1020. How much was invested at each rate?
Read: We are told that Julie invested money in two different places: one at 4.5% simple interest and one at 3% simple interest. (Recall that simple interest is computed by multiplying the amount invested (called the principal) by the interest rate.) The problem also tells us that her total interest earned was \$1020. We are asked to find the amount invested in each of the two accounts.
Assign a variable: The unknowns here are the amounts invested in each of the accounts. The amount invested in the 4.5% account is completely unknown, so let's call that x. We are given a formula for the amount invested in the 3% account: $2x - 1000$ ("\$1000 less than twice the amount invested in the 4.5% account").
Write an equation: To construct our equation, let's first fill out a table.

Rate	**Principal**	**Interest**
.045	x	$.045x$
.03	$2x - 1000$	$.03(2x - 1000)$
		1020

The values $.045x$ and $.03(2x - 1000)$ represent the amount of interest earned from their respective accounts and come from the fact that we know how to compute interest (interest rate times principal). They should sum to the total interest earned (\$1020). This leads us to

$$.045x + .03(2x - 1000) = 1020$$

as our equation.
Solve:

$$\begin{aligned}
.045x + .03(2x - 1000) &= 1020 && \text{(original equation)} \\
.045x + .06x - 30 &= 1020 && \text{(distributive property)} \\
.105x - 30 &= 1020 && \text{(combine like terms)} \\
.105x - 30 + 30 &= 1020 + 30 && \text{(cancel the 30)} \\
.105x &= 1050 \\
\tfrac{.105x}{.105} &= \tfrac{1050}{.105} && \text{(isolate the } x\text{)} \\
x &= 10000
\end{aligned}$$

Our solution for x tells us that the amount invested in the 4.5% account is \$10,000. To find the amount invested in the 3% account, we use the formula from the problem: $2x - 1000$. Substituting 10000 for x in this equation, we find that $2(10000) - 1000 = 20000 - 1000 = 19000$. So, \$19,000 was invested in the 3% account.

State the answer: Jane invested \$10,000 in the 4.5% account and \$19,000 in the 3% account. As far as we can tell, these answers are plausible. As before, negative numbers or extremely large or small answers are a clue that something went wrong.

Check: We can plug our value for x back into our original equation to make sure everything comes out right.

$$\begin{aligned}
.045x + .03(2x - 1000) &= .045(10000) + .03(2(10000) - 1000) \\
&= .045(10000) + .03(20000 - 1000) \\
&= .045(10000) + .03(19000) \\
&= 450 + 570 \\
&= 1020
\end{aligned}$$

This agrees with the information provided in the problem, so we know our answer is right.

2.3.49

Ten liters of a 4% acid solution must be mixed with a 10% solution to get a 6% solution. How many liters of the 10% solution are needed?

Read: The problem tells us that we already have 10 liters of a 4% acid solution in a bucket. We need to add some 10% acid solution until the overall concentration of acid in the bucket is 6%. The problem is to find out exactly how much of the 10% acid solution we will have to add. Note that to determine the liters of pure acid in a given solution, we multiply the percent concentration by the total liters of solution.

Assign a variable: The unknown quantity is the amount of 10% acid solution that we are going to add. Let's call this amount x.

Write an equation: As before, let's work out a table to organize our information.

Percent	**L Solution**	**L Pure Acid**
.04	10	.4
.1	x	$.1x$
	$10 + x$	$.06(10 + x)$

The last line of the table is a little mysterious. The value $10 + x$ represents the total liters of solution when we're all finished mixing. Since we started with 10L and we are adding xL more, we end up with a total of $10 + x$L. The value $.06(10 + x)$ means "$(10 + x)$L of 6% solution", which is what we hope to end up with when we're finished. Since we are physically *adding* the 4% solution and the 6% solution together, we should suspect that our equation is

$$.4 + .1x = .06(10 + x).$$

Solve: It is left as an exercise to find that $x = 5$.
State the answer: We must add 5 liters of the 10% acid solution to acheive an overall concentration of 6%. Our answer doesn't seem absurd (negative, tiny, or huge), so it's worth keeping around to check. While it isn't explicitly asked for in the problem, it's worth noting that we now have a total of 15 liters of solution.
Check: Substitute 5 for x in our original equation.

$$\begin{aligned} .4 + .1(5) &= .06(10 + (5)) \\ .9 &= .9 \end{aligned}$$

Since we get a true statement ($.9 = .9$), we know our answer of $x = 5$ is right.

2.4.23
Latrella can get to school in 15 min if she rides her bike. It takes her 45 min if she walks. Her speed when walking is 10 mph slower than her speed when riding. What is her speed when she rides?
Read: This problem is significantly more convoluted than the previous problems, but not any harder to solve. The first "red flag" is that time is given in *minutes* and speed is given in miles per *hour*. We cannot simply mix the two. So, we'll turn 15 min into $\frac{1}{4}$ hour and 45 min into $\frac{3}{4}$ hour. Now, we are given the time it takes Latrella to travel from home to school both by walking and by biking. We are also told something about her traveling speeds: she walks 10 mph slower than she rides. We are asked to find her riding speed. (Recall that distance = rate * time, or $d = rt$.)
Assign a variable: The unknown here is her riding speed. Let's call this x. This immediately induces a formula for her walking speed: $x - 10$.
Write an equation: We form the following table.

	Rate	**Time**	**Distance**
Riding	x	$\frac{1}{4}$	$\frac{1}{4}x$
Walking	$x - 10$	$\frac{3}{4}$	$\frac{3}{4}(x - 10)$

We are tempted by habit here to add the entries in the rightmost column together and set it equal to some total. This problem has no total or end-result we're trying to reach, so we'll have to try something else. We know that the distance between home and school is the same regardless of whether she rides or walks (or drives or swims or takes a plane or...), so we use the following equation

$$\frac{1}{4}x = \frac{3}{4}(x - 10)$$

Solve: It is left as an exercise to find that $x = 15$.
State the answer: Latrella's riding speed is 15mph. We also know that her walking speed is 15 - 10 = 5 mph. Both seem reasonable, so we check to be sure.
Check: Substitute $x = 15$ into our original equation.

$$\begin{aligned} \frac{1}{4}(15) &= \frac{3}{4}(15 - 10) \\ \frac{15}{4} &= \frac{15}{4} \end{aligned}$$

Since we get a true statement ($\frac{15}{4} = \frac{15}{4}$), we know our answer of $x = 15$ is right.

Final Remarks When solving word problems, keep in mind the six step outline we used throughout this guide. Carefully craft your table (or whatever method you use to organize your information) and call on outside knowledge to determine relationships between the given information (such as $d = rt$). Make sure you use all the given information (there won't be any trick questions with

useless information). Keep a close eye on units: you can't combine hours and minutes. When you have an answer, ask yourself what it means and whether it makes sense. Finally, always check your answer by substituting it back into the original equation.

Optional Assignment

The following assignment is worth 10 points and is due by Friday, 9/14 (the day of Exam 1). If you choose to turn it in, I will record either the grade for this assignment or assignments 3 and 4 (whichever is better).

2.3.30, 2.3.46, 2.3.52, 2.4.8, 2.4.20, 2.4.21, 2.2.24