

### Problems on Ramsey Theory

1. Prove that  $R(2, n) = n$  for any  $n$ .  
Hint: Can you even use red?
2. Prove that  $R(3, 5) \leq 14$ .  
Hint:  $R(2, 5) = 5$  and  $R(3, 4) = 9$ .
3. Prove that any coloring of the edges of  $K_6$  with two colors contains at least two monochromatic  $K_3$ 's. (The  $K_3$ 's are allowed to have some (but not all) vertices and edges in common. It is also permissible for one of the  $K_3$ 's to be red and the other blue.)  
Hint: Since  $R(3, 3) = 6$ , you already know that you will get one monochromatic  $K_3$ . When you try to put in the remaining edges while avoiding a second monochromatic  $K_3$ , you'll find out that it can't be done.