

Core Problems

Bring a printed draft to class on the specified due date. Your revised final version is due two weeks after the draft.

1. (**Draft: February 1**) Let $a_0 = a_1 = 1$, and let $a_n = a_{n-1} + 5a_{n-2}$ for $n \geq 2$. Prove that $a_n \leq 3^n$ for all $n \geq 0$. (Hint: Use strong induction on n .)
2. (**Draft: February 8**) Prove that for all positive integers n ,

$$n \binom{2n-1}{n-1} = \sum_{k=1}^n k \binom{n}{k}^2.$$

(Hint: Imagine you are forming a committee of n people from a group of n men and n women. How can you interpret each side of the equation combinatorially?)

3. (**Draft: February 15**) Let $F(n, k)$ be the number of partitions of the set $[n]$ into exactly k blocks in which each block contains two or more elements. Express the numbers $F(n, k)$ in terms of the Stirling numbers of the second kind. (Hint: For $1 \leq i \leq n$, define A_i to be the collection of partitions in which element i is a singleton. The set $A_1 \cup \dots \cup A_n$ therefore contains all “bad” partitions.)

Individual Problems

Complete only the problem assigned to you. Bring three printed drafts to class on **February 22**. Critiques of the drafts are due **March 1**, and your revised final version is due **March 8**.

- Kenzie: Let $F(n)$ be the number of set partitions having no singleton block. Construct a bijection to prove $B(n) = F(n) + F(n+1)$. (Hint: Prove instead that $B(n) - F(n) = F(n+1)$. What sorts of partitions are described by each side of the equation?)
- Megan: Let a_n be the number of compositions of n into parts that are 2 or larger. Describe a recurrence for a_n in terms of a_{n-1} and a_{n-2} . (Hint: Consider the possibilities for the first element of a composition of n into parts that are 2 or larger.)
- Yesenia: Prove the inequality $p(n)^2 < p(n^2 + 2n)$ for all positive integers n . (Hint: Read about Durfee squares on page 105, Exercise 8. Let P_n be the collection of partitions of n . Construct an injection from $P_n \times P_n$ into P_{n^2+2n} .)
- Carter: Find a closed formula for $S(n, n-3)$ for all $n \geq 3$. (Hint: What are the possibilities for singleton and non-singleton blocks?)