Updated April 4, 2019.

Instructions

Pre-class assignments are intended to help you get more out of class. You are not expected to provide a complete solution, but you should give it your best attempt and try to reflect on some of the big ideas. If you are not confident in your solution, describe some *specific* points where your understanding could be improved. Write out all your work and questions and submit them in writing (not in LaTeX) at the beginning of class on the due date.

January 23

Section 1.1

- No pre-class homework.

January 25

Section 1.2

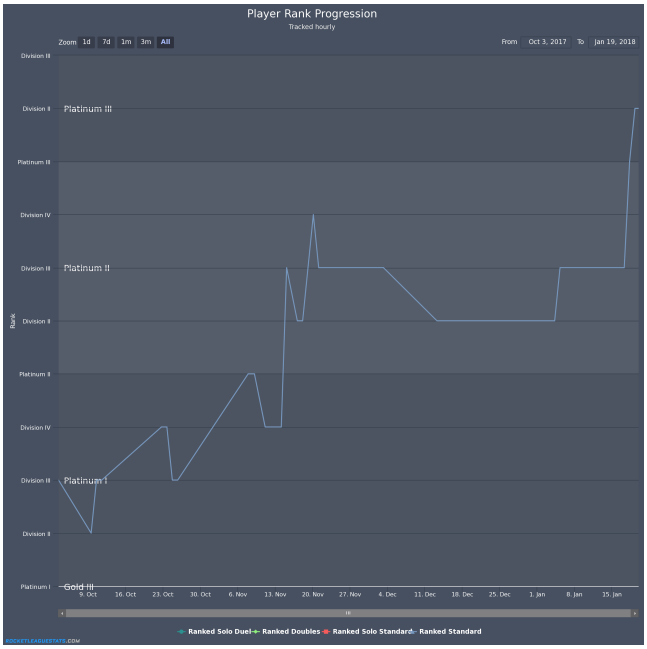
- Optional viewing: Watch a video on making rigorous definitions.
  https://www.youtube.com/watch?v=RoqoSxUoK-A
  - The important bit from this video is the definition of an even integer: An integer n is said to be **even** if there exists another integer k such that $n = 2k$.
- Watch a video on know-show tables.
  https://www.youtube.com/watch?v=H8LLINU6ebY
- Construct a know-show table for the following conditional statement:
  If m and n are even integers, then mn is an even integer.

January 28

Section 2.1

- Read the definitions of conjunction, disjunction, negation, and implication on page 33 in the text, and take a look at the truth tables for these operations on page 36.
- Watch a video on using truth tables to analyze compound statements.
  https://www.youtube.com/watch?v=d2yksDk4h6s

- Construct a truth table for the following compound statement: I played Rocket League for six consecutive hours and I did not reach Diamond rank.



January 30

Section 2.2

- Watch a video on logical equivalence.
https://www.youtube.com/watch?v=oY8Xt5GvZ1g
- Use truth tables to prove the following two logical equivalences. (These are very useful equivalences known as DeMorgan's Laws.)

$$\neg(P \vee Q) \equiv (\neg P) \wedge (\neg Q)$$
$$\neg(P \wedge Q) \equiv (\neg P) \vee (\neg Q)$$

February 1

Section 2.3

- Optional viewing: Watch a video introducing the concept of a set.
https://www.youtube.com/watch?v=01OoCH-2UWc
- Watch a video introducting set builder notation.
https://www.youtube.com/watch?v=DEhWj-52rlw
- What elements belong to the set $\{n \in \mathbb{Z} \mid n^2 < 16\}$?
- Use set builder notation to describe the set $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$.

February 4

Section 2.4

- Watch a video introducing universal and existential quantifiers.
  https://www.youtube.com/watch?v=6qTzP03waOA
- Write each of the following statements in English and explain why the statement is false.

$$(\exists x \in \mathbb{R})(x^2 < 0)$$

$$(\forall n \in \mathbb{Z})(n^2 \geq 1)$$

February 6

Section 3.1

- Watch a video about integer divisibility.
  https://www.youtube.com/watch?v=dIfpZzX7bKo
- Show that each of the following statements is true.
  - $2 \mid 10$
  - $5 \mid 15$
  - $3 \nmid 8$

February 8

Section 3.1

- Watch a video about integer congruence.
  https://www.youtube.com/watch?v=_-ZMdsQyIJw
- Show that each of the following statements is true.
  - $6 \equiv 0 \pmod 3$
  - $6 \equiv 11 \pmod 5$
  - $6 \not\equiv 11 \pmod 4$

February 11

Section 3.2

- Watch a video about proof by contraposition.
  https://www.youtube.com/watch?v=hAFpc9abNFc

- Use contraposition to sketch a proof for the following proposition. (You need not write in full detail.)
  If n^3 is even, then n is even.

February 13

Section 3.3

- Watch an introduction to proof by contradiction.
  https://www.youtube.com/watch?v=YUL6HMJmTM4
- Sketch a proof by contradiction for the following statement: There are no integers m and n such that $6m + 15n = 2$. (Hint: Factor a common 3 from the lefthand side.)

February 15

Section 3.3

No pre-class homework.

February 18

Section 3.4

- Watch an introduction to proof by cases.
  https://www.youtube.com/watch?v=zmMk_YITBIo
- Sketch a proof by cases for the following statement: If a is even or b is even, then ab is even.

February 20

Class cancelled.

February 22

Section 3.5

- Watch a video on the division algorithm.
  https://www.youtube.com/watch?v=XHjSy_MT7u0
- Rewrite the expression $23 \div 4$ in the $a = bq + r$ form specified by the division algorithm.

February 25

Review for Exam 1

No pre-class homework.

February 27

Exam 1

No pre-class homework. Exam 1 will feature problems 1 – 4.

March 1

Section 4.1

- Watch an introduction to mathematical induction.
  - Part 1: https://www.youtube.com/watch?v=JTj6ID4-084
  - Part 2: https://www.youtube.com/watch?v=1H0gg3fMYVA
- Carefully state the inductive hypothesis for the following proposition. (You do not have to prove it.)

  For every natural number n, it is true that

$$1 + 2 + \cdots + n = \frac{1}{2}n(n+1).$$

March 4

Section 4.1

No pre-class homework.

March 6

Section 4.2

- Watch a video on the second principle of induction (often called "strong induction"). https://www.youtube.com/watch?v=n-bJB_7QbQU

- Carefully state the strong inductive hypothesis for the following proposition. (You do not have to prove it.)

  Each natural number greater than or equal to 4 can be written as a sum of 2's and 3's. (For example, $12 = 2 + 2 + 2 + 3 + 3$.)

March 8

Section 4.3

- Watch an introduction to recursively-defined sequences.
  https://www.youtube.com/watch?v=txdmCgThR6o
- Write out the first five terms of the following recursively-defined sequence:
  $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$ and $a_0 = a_1 = 1$

March 18

Section 5.1

- Watch an introduction to set operations.
  https://www.youtube.com/watch?v=QiOfsWm3peE
- Define the sets $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ viewed as subsets of the universal set $\mathbb{N}$. Write down the contents of the sets $A \cup B$, $A \cap B$, $A - B$, and A^c.

March 20

Section 5.2

- Watch an introduction to subset inclusion ("element-chasing") proofs.
  https://www.youtube.com/watch?v=8QmqjwFLV7k
- Let A contain all positive multiples of 6 ($A = \{6, 12, 18, \dots\}$) and let B contain all positive multiples of 3 ($B = \{3, 6, 9, \dots\}$). Sketch a proof that $A \subset B$.

March 22

Exam 2a

No pre-class homework. Exam 2a will feature problems 1 – 4.

March 25

Section 5.3

- Watch a discussion on various set identities.
  https://www.youtube.com/watch?v=gUMmcfyGb-U
- Sketch a proof that $(A \cap B)^c = A^c \cup B^c$ for all sets A and B.

March 27

Section 5.3

No pre-class homework.

March 29

Section 5.4

- Watch an introduction to Cartesian products.
  https://www.youtube.com/watch?v=knwM9OWK3oA
- Let $A = \{a, b, c\}$ and $B = \{1, 2\}$. Write out all the elements of $A \times B$.

April 1

Section 6.1

- Watch an introduction to functions.
  https://www.youtube.com/watch?v=PZFU3JC3Plo
- Refer to the "five basic ingredients" in the video. Suppose I create a function that, presented with any person in the class, returns the color of their eyes. Discuss the five basic ingredients as they pertain to this function. Why does our function break if any of us happen to have two colors of eyes?

April 3

Section 6.3

- Watch an introduction to the concept of injectivity. (You can stop watching at the "Concept Check" if you like.)
  https://www.youtube.com/watch?v=fGYaaKryZp4

- Watch an introduction to the concept of surjectivity. (You can stop watching at the "Concept Check" if you like.)
  https://www.youtube.com/watch?v=jVniPMIexQE
- Suppose I create a function that, presented with any person in the class, returns their favorite movie. What would it mean for this function to be injective? Why is it impossible for this function to be surjective?

April 5

Section 6.4

- Watch an introduction to composition of functions.
  https://www.youtube.com/watch?v=f_t1I3WgXbM
- Define functions $f : \mathbb{N} \to \mathbb{N}$ by $f(x) = 3x+1$ and $g : \mathbb{N} \to= \mathbb{R}$ by $g(x) = \sqrt{x}$. Evaluate each of the following or explain why it does not exist.
  - $(g \circ f)(2)$
  - $(f \circ g)(2)$

April 8

Section 6.5

- Watch an introduction to inverses of functions.
  https://www.youtube.com/watch?v=EpmgqJ9Xog0
- Define $f : \{0, 1, 2\} \to \{0, 1, 2\}$ by $f(x) = x + 2 \pmod 3$. Express both f and f^{-1} as a set of ordered pairs.

April 10

Section 7.1, 7.2

- Watch an introduction to relations.
  https://www.youtube.com/watch?v=qnjxdlpWMLA
- Recall that $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$. Define the relation $R = \{(m, n) \in \mathbb{Z}_6 \times \mathbb{Z}_6 \mid m \equiv n \pmod 3\}$. Write out all ordered pairs that belong to R.

April 12

Review for Exam 2b

We will meet in Smith-Curtis 122 today.

No pre-class homework. Exam 2b will feature questions 5 – 7 from the study guide.

April 15

Exam 2b

No pre-class homework. Exam 2b will feature questions 5 – 7 from the study guide.

April 17

Section 7.3

- Watch an introduction to equivalence classes.
  https://www.youtube.com/watch?v=-C6Rnk0W2lE
- Define the relation $R = \{(m, n) \in \mathbb{Z} \times \mathbb{Z} \mid m \equiv n \pmod 2\}$, which is an equivalence relation. Describe the two equivalence classes induced by R.

April 24

Section 9.2

- Watch an introduction to countably infinite sets.
  https://www.youtube.com/watch?v=nku7KxZrjmk
- Let D^+ denote the set of odd natural numbers. Sketch a proof that D^+ is countably infinite by first constructing a function $f : \mathbb{N} \to D^+$ with the appropriate properties.

April 26

Exam 3

No pre-class homework. Exam 3 will feature questions 1 – 9 from the study guide.

April 29

Section 9.3

No pre-class homework.

May 1

NWU Student Research Symposium

No class. You may write brief reflections on **mathematics** presentations for extra credit toward your post-class homework. Each reflection is worth one point of extra credit, and you may submit as many as you like.

May 3

Review for Final

No required meeting, but consider using this time to study for the final with your groupmates.

May 8

Final Exam

The final exam starts at 8 am in our usual classroom. It will feature questions 1 – 9.