Updated April 10, 2019.

Instructions

Post-class assignments are intended to help you clarify your understanding of the big ideas in the course. They will be graded on a rough five-point scale. You are welcome to collaborate with classmates or visit me during office hours, but your final submission must be your own work. Typeset your solution in LaTeX unless stated otherwise. Submit both your pdf and tex files via email by 11:59 pm on the due date with [2200] and the problem number in the subject line (e.g., [2200] Problem 1).

Problem 1 (Due January 25)

Download the file "Post-Class Problem 1" from the course website. Use LaTeXto recreate its contents exactly (with the exception of updating the name and date).

Problem 2 (Due January 30)

Let a, b, and c be an odd integers. Prove that $a + bc$ is an even integer.

Problem 3 (Due February 1)

(You may submit this problem in writing rather than in LaTeX.)
Complete the following exercises from the end of Section 2.1 in the text: 5d, 6a, and all parts of 9.

Problem 4 (Due February 4)

(You may submit this problem in writing rather than in LaTeX.)

Definition. *We say an integer k **divides** another integer n provided we can write $n = km$ for some integer m. (For example, 3 divides 15 since $15 = 3 \cdot 5$.)*

Part A

The following proposition is true. You do not need to prove it.

Proposition. *If a divides b and a divides c, then a divides $b + c$.*

Write the inverse, converse, and contrapositive of this proposition. (Don't forget to apply DeMorgan's Law.) Which of these is equivalent to the original proposition? For the others, provide a specific example of a, b, and c where the statement fails to be true.

Part B

Show that the following statement is not true in general by writing its negation and giving a specific example where the negation is true.

> If a divides bc, then a divides b or a divides c.

Problem 5 (Due February 6)

(You may submit this problem in writing rather than in LaTeX.)
Complete the following exercises from the end of Section 2.3 in the text: 5de and 6ce

Problem 6 (Due February 8)

(You may submit this problem in writing rather than in LaTeX.)
Complete the following exercises from the end of Section 2.4 in the text: 3ab and all parts of 8

Problem 7 (Due February 15)

Prove the following statement by contraposition (see exercise # 9 from Section 3.2 in the text for more details):

> Let $x \in \mathbb{R}$. If x is irrational, then $\sqrt{x}$ is irrational.

Problem 8 (Due February 22)

Prove the following statement by contradiction.

> If x is irrational and y is any real number, then $x + y$ is irrational or $x - y$ is irrational.

Problem 9 (Due March 1)

Prove the following statement by case analysis.

> For each integer n, if $n \not\equiv 0 \pmod 3$, then $n^2 \not\equiv 0 \pmod 3$.

Problem 10 (Due March 6)

Prove the following statement by induction.

> The congruence $4^n \equiv 1 \pmod 3$ holds for each natural number n.

Problem 11 (Due March 20)

Let $a_1 = a_2 = a_3 = 1$ and define $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for each natural number $n \geq 4$. Prove (by strong induction) that $a_n \leq 2^{n-2}$ for each natural number $n \geq 2$.

Problem 12 (Due March 27)

Use element-chasing to prove that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ for all sets A, B, and C.

Problem 13 (Due April 1)

Use set identities to prove that $A - (B \cup C) = (A - B) \cap (A - C)$ for all sets A, B, and C.

Problem 14 (Due April 10)

Prove that if $f : A \to B$ and $g : B \to C$ are injective, then $(g \circ f) : A \to C$ is injective. (Hint: Mimic the proof of Theorem 6.20.)

Problem 15 (Due April 12)

Prove that if $f : A \to B$ is bijective, then $f^{-1} : B \to A$ is bijective. (You may assume without proof that f^{-1} is a function, since this is proven in Theorem 6.25 in the text.)

Problem 16 (Due April 24)

Define the relation $R = \{(m, n) \in \mathbb{N} \times \mathbb{N} \mid m + n \equiv 0 \pmod 3\}$. Decide whether R is reflexive, symmetric, and/or transitive. Give a proof or counterexample in each case.

Problem 17 (Due April 29)

Prove that if A and B are countably infinite sets, then $A \cup B$ is a countably infinite set. (See Theorem 9.17 in the text for a hint.)