Proof Portfolio # 7

Austin Mohr

April 15, 2019

Proposition. *For any natural number n, there are as many subsets of $[n]$ as there are binary strings of length n.*

Proof. Let $n \in \mathbb{N}$ be given and let A_n denote the collection of all binary strings of length n. Define a function $f : \mathcal{P}([n]) \to A_n$ in the following way: Given a subset S of $[n]$, define $f(S)$ to be the binary string whose k^{th} digit is 1 if $k \in S$ and 0 if $k \notin S$ (for $1 \leq k \leq n$). We claim that f is a bijection.

To show that f is injective, begin by assuming that $S_1 \neq S_2$, where S_1 and S_2 are subsets of $[n]$. We want to show that $f(S_1) \neq f(S_2)$. **(Your assumption is that the two sets differ, so one of S_1 or S_2 has an element that the other does not. Use this to conclude that the associated binary strings also differ.)**

To show that f is surjective, let us be given a binary string a of length n. We want to show that there is a subset S of $[n]$ such that $f(S) = a$. **(Show that there is a set S such that $f(S) = a$ by describing how to reconstruct S from its binary string representation.)** □