Updated February 22, 2019

Proof Portfolio

Description

The proof portfolio requires you to write and revise eight short proofs of statements drawn from the course material. All proofs must be written in LaTeX. Refer to the provided rubric throughout the process to help direct your writing. You will also critique the writing of your peers to help improve their writing as well as your own.

Each problem has three stages: Draft, Reflection, and Final.

- Draft: Make your best effort at a completed proof. This version will not affect your grade, but will help guide you in the writing of a quality final draft.
- Reflection: You will reflect on the writing of two of your classmates. Refer to the four categories in the writing rubric: Correctness, Rigor, Clarity, and Formatting. Compare and contrast your work with the work of your peers in relation to each of these four categories. What did your peers do more effectively than you? What did you do more effectively than your peers? What kinds of changes will you make for your final draft? Summarize your thoughts by writing a few sentences for each category.
- Final: Revise your first draft according to the feedback you received from me and the reflection you wrote for yourself. I will grade this final draft according to the posted rubric.

Timeline

- Draft: Bring three printed copies of the specified problem to class.
- Reflection: Submit your written reflection to me either in hard copy or by email (with a subject line like "[2200] Reflection 1").
- Final: Email your final proof to me (pdf and tex) with a subject line like "[2200] Portfolio 1".

Date	**Draft**	**Reflection**	**Final**
2-22	1		
3-1	2	1	
3-8	3	2	
3-15			
3-22	4	3	
3-29	5	4	
4-5	6	5	
4-12	7	6	1, 2
4-17*	8	7	3, 4
4-26		8	5, 6
5-3			7, 8

* Note that April 17 is a Wednesday.

Problems

1. Prove that the equation $n^7 - 3n^4 - 9n - 7 = 0$ has no natural number solution.

2. Let a and b be integers. Prove that if $a^2 + 2b^2 \equiv 0 \pmod 3$, then either a and b are both congruent to 0 modulo 3 or neither are congruent to 0 modulo 3.

3. Let a and b be integers and let p be prime. Prove that if $a^2 \equiv b^2 \pmod p$, then $a \equiv b \pmod p$ or $a \equiv -b \pmod p$. (You may use the following lemma without proof: Let p be prime and let a and b be any integers. If $p \mid ab$, then $p \mid a$ or $p \mid b$.)

4. Prove that $|x + y| \leq |x| + |y|$ for all real numbers x and y.

5. Prove that $\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{7}{6}n$ is a natural number for all natural numbers n.

6. Prove that $A - (B - C) = (A \cap C) \cup (A - B)$ for all sets A, B, and C.

7. Let $n \in \mathbb{N}$ and let $[n]$ denote the set $\{k \in \mathbb{N} \mid 1 \leq k \leq n\}$. Prove that there are as many subsets of $[n]$ as there are binary strings of length n. (Hint: Establish a bijection between these two collections.)

8. For $a, b \in \mathbb{Z}$, define $a \sim b$ to mean $2a + 3b \equiv 0 \pmod 5$. Prove that $\sim$ is an equivalence relation.