Updated April 29, 2019

Proof Portfolio

Description

The proof portfolio requires you to write and revise eight short proofs of statements drawn from the course material. All proofs must be written in LaTeX. Refer to the provided rubric throughout the process to help direct your writing. You will also critique the writing of your peers to help improve their writing as well as your own.

Each problem has three stages: Draft, Reflection, and Final.

- Draft: Make your best effort at a completed proof. This version will not affect your grade, but will help guide you in the writing of a quality final draft.

- Reflection: You will reflect on the writing of two of your classmates. Refer to the four categories in the writing rubric: Correctness, Rigor, Clarity, and Formatting. Compare and contrast your work with the work of your peers in relation to each of these four categories. What did your peers do more effectively than you? What did you do more effectively than your peers? What kinds of changes will you make for your final draft? Summarize your thoughts by writing a few sentences for each category and email the reflection to me. You do not need to return any feedback to the individuals whose proofs you reviewed.

- Final: Revise your first draft according to the feedback you received from me and the reflection you wrote for yourself. I will grade this final draft according to the posted rubric.

Timeline

- Draft: Bring three printed copies of the specified problem to class.

- Reflection: Submit your written reflection to me either in hard copy or by email (with a subject line like "[2200] Reflection 1").

- Final: Email your final proof to me (pdf and tex) with a subject line like "[2200] Portfolio 1".

Date	Draft	Reflection	Final
2-22	1		
3-1	2	1	
3-8	3	2	
3-15			
3-22			
3-29	4, 5	3	
4-5	6	4, 5	
4-12		6	1, 2
4-17*	7, 8		3, 4
4-26		7, 8	5, 6
5-3		Overall	7, 8

* Note that April 17 is a Wednesday.

Final Submission

At the end of the course, please email me a **single** document (both the tex and pdf files) containing

- all eight portfolio problem final drafts and

- your final, overall reflection (not the individual eight short reflections).

Problems

1. Prove that the equation $n^7 - 3n^4 - 9n - 7 = 0$ has no natural number solution.

2. Let a and b be integers. Prove that if $a^2 + 2b^2 \equiv 0 \pmod 3$, then either a and b are both congruent to 0 modulo 3 or neither are congruent to 0 modulo 3.

3. Let a and b be integers and let p be prime. Prove that if $a^2 \equiv b^2 \pmod{p}$, then $a \equiv b \pmod{p}$ or $a \equiv -b \pmod{p}$. (You may use the following lemma without proof: Let p be prime and let a and b be any integers. If $p \mid ab$, then $p \mid a$ or $p \mid b$.)

4. Prove that $|x + y| \leq |x| + |y|$ for all real numbers x and y.

5. Prove that $\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{7}{6}n$ is a natural number for all natural numbers n.

6. Prove that $A - (B - C) = (A \cap C) \cup (A - B)$ for all sets A, B, and C.

7. Let $n \in \mathbb{N}$ and let $[n]$ denote the set $\{k \in \mathbb{N} \mid 1 \leq k \leq n\}$. Prove that there are as many subsets of $[n]$ as there are binary strings of length n. (Hint: Establish a bijection between these two collections.)

8. For $a, b \in \mathbb{Z}$, define $a \sim b$ to mean $2a + 3b \equiv 0 \pmod 5$. Prove that $\sim$ is an equivalence relation.

Overall Reflection

Once you have completed final drafts for all the problems, look back on all your old reflections and drafts. Write a final, overall reflection that reflects on the overall trajectory of your writing through the course. It should be $2 - 3$ pages in length and focus on our same criteria of Correctness, Rigor, Clarity, and Formatting. I don't want to constrain your writing by giving too many rules, but here are some questions that might help you get started:

- What are some instances in the course where it was difficult to find a correct mathematical proof? What are some habits that have helped you to reliably detect incorrect statements? How do you go about correcting them?

- What is rigor, as it pertains to mathematical writing? Why is it important? How have you tried to achieve it in your writing, and how has that effort changed over time?

- What are some things that make mathematical writing clear or unclear? Have these habits shown up in your writing? How has your writing changed in terms of clarity over time? How can clarity and rigor sometimes be at odds with one another?

- What have you learned about formatting mathematics in LaTeX? How do you think formatting has increased or inhibited clarity in the past? How have you developed in your formatting skill over time?