Name: ______________________________

Please write *only* your name on the test sheet.
Place all work and answers on the blank sheets provided.
Only attempt problems that you have not previously mastered.

1. (a) Write the negation of the following proposition in English and provide values of the variables for which the negation is true.
   Let an integer n be given. If $2 \mid n$ and $3 \mid n$, then $n = 0$ or $n = 6$.

   (b) Express the following statement and its negation symbolically.
   There exists a real number x such that, for all real numbers y, both $\frac{x}{y}$ and $\frac{y}{x}$ are rational.

2. Prove the following directly: If x and y are rational numbers, then $x + y$ is a rational number.

3. Prove the following by contraposition: For all integers n, if n^2 is odd, then n is odd.

4. Prove the following by contradiction: If x is rational and y is irrational, then $x + y$ is irrational.

5. Use induction to prove the following identity for all natural numbers n:

$$\sum_{k=1}^{n}(2k-1) = n^2.$$

6. Prove that $(A \cup B) - C \subseteq (A - C) \cup (B - C)$ for all sets A, B, and C.

7. Let Y denote the set of odd integers. Define the function $f : \mathbb{Z} \to Y$ by $f(n) = 2n + 11$. Prove that f is a bijection.

8. Define a relation $\sim$ on $\mathbb{Z}$ as follows: For $m, n \in \mathbb{Z}$, write $m \sim n$ to mean $2 \mid (m + n)$. Show that $\sim$ is an equivalence relation.

9. Let $A = \{\frac{m}{n} \in \mathbb{Q} \mid m \in \{-1, 1\}, n \in \mathbb{N}\}$. Prove that A is countable by establishing an injection into the natural numbers. (You may not appeal to any theorems on countability.)