

# Example Proofs

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**Definition.** *An integer  $n$  is an odd integer provided there exists an integer  $k$  such that  $n = 2k + 1$ .*

**Proposition.** *If  $n$  is odd, then  $n^3$  is odd.*

*Proof.* Since  $n$  is odd, we may write  $n = 2k + 1$  for some integer  $k$ . We aim to show that  $n^3$  is odd, so we consider

$$\begin{aligned}n^3 &= (2k + 1)^3 \\ &= 8k^3 + 12k^2 + 6k + 1 \\ &= 2(4k^3 + 6k^2 + 3k) + 1.\end{aligned}$$

By the closure of the integers under addition and multiplication, we know that  $4k^3 + 6k^2 + 3k$  is an integer. Call this integer  $m$ , so that we have  $n^3 = 2m + 1$ . Therefore,  $n^3$  is odd.  $\square$

**Definition.** *A nonzero integer  $m$  divides an integer  $n$  provided there is an integer  $k$  such that  $n = mk$ .*

**Proposition.** *Let  $a, b, c \in \mathbb{Z}$  with  $a \neq 0$ . If  $a \mid b$  and  $a \mid c$ , then  $a \mid (b - c)$ .*

*Proof.* Since  $a \mid b$  and  $a \mid c$ , we may write  $b = aj$  and  $c = ak$  for some  $j, k \in \mathbb{Z}$ . It follows that

$$\begin{aligned}b - c &= aj - ak \\ &= a(j - k).\end{aligned}$$

The integers are closed under subtraction, so  $j - k$  is some integer  $m$ . Thus, we have shown  $b - c = am$ , which is to say  $a \mid (b - c)$ .  $\square$