Updated April 16, 2018.
(Problem 14 is the last one, I promise.)

Instructions

Post-class assignments are intended to help you clarify your understanding of the big ideas in the course. They will be graded on a rough five-point scale. You are welcome to collaborate with classmates or visit me during office hours, but your final submission must be your own work. Typeset your solution in LaTeX unless stated otherwise. Submit both your pdf and tex files via email by 11:59 pm on the due date with [2200] and the problem number in the subject line (e.g., [2200] Problem 1).

Problem 1 (Due January 26)

Let a be any integer and let b and c be odd integers. Prove that $ab + ac$ is an even integer.

Problem 2 (Due January 31)

(You may submit this problem in writing rather than in LaTeX.)

Definition. *We say an integer k **divides** another integer n provided we can write $n = km$ for some integer m. (For example, 3 divides 15 since $15 = 3 \cdot 5$.)*

Part A

The following proposition is true. You do not need to prove it.

Proposition. *If a divides b and a divides c, then a divides $b + c$.*

Write the inverse, converse, and contrapositive of this proposition. Which of these is equivalent to the original proposition? For the others, provide a specific example of a, b, and c where the statement fails to be true.

Part B

Show that the following statement is not true in general by writing its negation and giving a specific exmaple where the negation is true.

If a divides bc, then a divides b or a divides c.

Problem 3 (Due February 5)

(You may submit this problem in writing rather than in LaTeX.)

Complete exercise # 11 from Section 2.4 in the text.

Problem 4 (Due February 12)

Prove the following statement by contraposition (see exercise # 9 from Section 3.2 in the text for more details):

Let $x \in \mathbb{R}$. If x is irrational, then $\sqrt{x}$ is irrational.

Problem 5 (Extended Due Date February 23)

Prove the following statement by construction:

Let $p, q \in \mathbb{Q}$ with $p < q$. There exists a rational number x such that $p < x < q$. (Hint: Let x be the average of p and q. Is x rational? Does it satisfy $p < x < q$?)

Problem 6 (Due February 21)

Prove the following statement by contradiction. (Hint: $\sin^2\theta + \cos^2\theta = 1$)

For each real number θ, if $0 < \theta < \frac{\pi}{2}$, then $\sin\theta + \cos\theta > 1$.

Problem 7 (Due February 21)

Prove the following statement by case analysis.

For each integer n, if $n \not\equiv 0 \pmod 7$, then $n^2 \not\equiv 0 \pmod 7$.

Problem 8 (Due March 19)

Prove the following statement by induction.

The congruence $4^n \equiv 1 \pmod 3$ holds for each natural number n.

Problem 9 (Due March 19)

Let $a_1 = a_2 = a_3 = 1$ and define $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for each natural number $n \geq 4$. Prove (by strong induction) that $a_n \leq 2^{n-2}$ for each natural number $n \geq 2$.

Problem 10 (Due March 26)

Prove that $(A \cap B) - C = (A - C) \cap (B - C)$ for all sets A, B, and C.

Problem 11 (Due April 11)

(You may submit this problem in writing rather than in LaTeX.)

Decide whether each of the following functions is injective and/or surjective. Justify your claims.

- $f : \mathbb{Z} \to \mathbb{Z}$
  $f(n) = 3n + 1$
- $g : \mathbb{R} \to (0, 1]$
  $g(x) = e^{-x^2}$

Problem 12 (Due April 16)

Let $f : X \to Y$ be a bijection. Prove both of the following.

- For each $x \in X$, the identity $(f^{-1} \circ f)(x) = x$ holds.
- For each $y \in Y$, the identity $(f \circ f^{-1})(y) = y$ holds.

Problem 13 (Due April 27)

Define the relation $R = \{(m, n) \in \mathbb{N} \times \mathbb{N} \mid m + n \text{ is a multiple of } 3\}$. Decide whether R is reflexive, symmetric, and/or transitive. Give a proof or counterexample in each case.

Problem 14 (Due April 27)

Prove that if A and B are countably infinite sets, then $A \cup B$ is a countably infinite set. (See Theorem 9.17 in the text for a hint.)