Nebraska Wesleyan University
Math 4800: Research Experience
Section 1
Spring 02016

Readings and Exercises

Tuesday, January 19

Section 1

- Exercise 1

Section 2

- Exercise 1, 4ce

Thursday, January 21

Section 3

- Read the definition of relation and Example 1.
- Read the definition of order relation and Example 7. Show the the relation defined at the end of Example 7 is indeed an order relation.
- Read everything on page 7 through the end of the section. Explain why the set B in Example 13 does not have the least upper bound property.
- Exercise 13

Section 5

- Read the definition of a finite cartesian product and Example 1. Using viewpoints of Example 1, give two descriptions of the cartesian product $\{0, 1\} \times \{0, 1\}$.
- Read the definition of an infinite cartesian product and Example 3.
- Exercise 4a, 4b, 4e

Tuesday, January 26

Section 7

- Read the definition countably infinite and Example 1. Why is the set of all integers countably infinite?
- Read the definition of countable, uncountable, and the statement of Theorem 7.1. Use the theorem to prove that the set of all even integers is countable. Could you use it to prove that any subset of a countable set is countable?
- Read Corollary 7.4. Extend the proof to show that $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$ is countably infinite. How would you adapt the proof for n copies of $\mathbb{Z}$?
- Read the statements of Theorem 7.5 and 7.6.
- Read the statement and proof of Theorem 7.7. Try to relate the proof to the usual presentation of Cantor's proof that the reals are uncountable by diagonalization.
- Read the statement and proof of Theorem 7.8. Elaborate, in your own words, about the subset B and the contradiction that arises.
- Exercise 3, 4, 5abefij

Thursday, January 28

Notice we will spend the following class period discussing exercises. Do not feel that you need to attempt every one for today.

Section 12

- Read the definition of topology and open set.
- The Sierpinski Two-Point Space is the simplest nontrivial topology. Take a look at the proof that it is indeed a topology on π-Base. http://topology.jdabbs.com/spaces/10
- Read Examples 1 through 4 very closely. The best way to get a handle on the abstract definition of topology is to see lots of examples.

Section 13

- Read the first paragraph and the definition of basis and topology generated by a basis.
- Explain why Example 1 satisfies the definition of a basis. (This basis is more or less the one used to generate the usual topology on the real plane.)
- Just after Example 3, the author explains why the topology generated by a basis is indeed a topology. Read this closely and try to summarize the discussion. (If this discussion feels too abstract, move on and come back to it at the end.)
- Read Lemma 13.1 and its proof. Notice how a basis lets one represent an open set as a very large (maybe uncountable) union of basis elements. You can use a similar idea in Exercise 1.
- Read the definition of the standard topology topology, lower-limit topology, and K-topology on the real line. Notice they are all distinct topologies on the same set.
- Exercise 1, 3, 6, 7, 8 (For 6 and 7, we say τ is contained in τ' if every open set of τ is also open in τ'. Two topologies are incomparable if neither contains the other.)

Tuesday, February 2

Snow day.

Thursday, February 4

Section 13

- Exercise 1, 3, 6, 7, 8 (For 6 and 7, we say τ is contained in τ' if every open set of τ is also open in τ'. Two topologies are incomparable if neither contains the other.)

Tuesday, February 9

Section 15

- Read the definition of a product topology. According to this definition, what are the basic open sets in the standard topology on $\mathbb{R}^2$? (See Example 1)
- Read the statement of Theorem 15.1. According to this theorem, what is a simpler basis for the standard topology on $\mathbb{R}^2$? (See Example 1)
- Draw a picture to show what's going on in the proof of Theorem 15.1 as it relates to the standard topology on $\mathbb{R}^2$. That is, given $(x, y) \in \mathbb{R}^2$ and an open set W containing it, how do we produce a basic open sets U and V such that $(x, y) \in U \times V \subset W$?

Section 16

- Read the definition of a subspace topology and Lemma 16.1. What are the basic open sets of $[0,1]$ as a subspace of the standard topology on $\mathbb{R}$? (See Example 1)
- Read the remark about distinguishing between "open in a space" and "open in a subspace". What are some sets that are open in the subspace on $[0,1]$ but not in $\mathbb{R}$?
- Exercise 1, 3, 6, 9

Thursday, February 11

Before our meeting, claim a "verify the topological axioms" proof to be included in your portfolio. You can claim a proof by following the link below and adding it under your name. You may choose any space that does not currently have a proof in π-Base, but I've included a few suggestions to get you started. Your pre-class assignment is to accomplish whatever you can toward a proof that the selected space is indeed a topology. Each of you will take 15 – 20 minutes during class to discuss your progress.

https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, February 16

Section 17

- Read the definition of closed set and the examples that follow. Give some examples of closed sets in the lower limit topology on $\mathbb{R}$.
- Read Theorem 17.1. What are the key differences between this theorem and the usual topological axioms?
- Read the definitions of interior and closure of a set. What is the interior of the set $[0,1]$ in the standard topology on the reals? What is the closure of the set $(0,1)$ in the same topology?
- Read Theorem 17.5 and its proof. Use this theorem to give an alternate argument for the closure of the set $(0,1)$ in the standard topology on the reals.
- Read the definition of neighborhood. (It's just useful terminology.)
- Read the definition of limit point, Example 8, and Theorem 17.6. Comment on the closure of $(0,1)$ in the standard topology on the reals. (In that example, what are the A, $\overline{A}$, and A' referred to in the theorem?)
- Read Corollary 17.7 and use it to prove that $[0,1]$ is closed in the standard topology on the reals.
- Read the whole introduction to the section on Hausdorff spaces, including the definition of convergence. (It's a nice exposition on an idea we've talked about before: Sometimes it's hard to pick just the right axioms to study.)
- Read the definition of Hausdorff space (also called the T_2 axiom). Prove that the standard topology on the reals is Hausdorff.
- Read Theorems 17.8 and 17.10 and their proofs. (There are very important properties about Hausdorff spaces.)
- Visit https://en.wikibooks.org/wiki/Topology/Separation_Axioms and read the definitions of the T_0 and T_1 axioms. Use π-Base to find some spaces that are T_0 but not T_1 and some spaces that are T_1 but not T_2. (The results of this search might be a good source of proofs for your portfolio.)
- Exercises 6, 10, 11, 12, 14, 16

Thursday, February 18

Please submit your reflection questions today.

Work on your portfolio prior to class and come prepared to present. You could present a more refined (preferably typeset) version of last week's proof, or you could choose to work on something new. If you want to choose something new, consider looking for proofs about the properties T_0, T_1, and T_2 to add to π-Base. Be sure to update the portfolio sheet accordingly.

https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, February 23

Section 18

- Read the definition of continuity and the following argument that we need only establish continuity on basic open sets.
- Read Example 1 and prove that the $\epsilon - \delta$ definition of continuity implies the more general definition for real-valued functions of a real variable.
- Read the statement of Theorem 18.1. It gives a few equivalent characterizations of continuity that could be useful.
- Read the definition of homeomorphism and the following paragraphs leading up to "topological property". Even though we won't make much use of it, the concept of a homeomorphism is one of the fundamental ideas in topology (and the source of the joke that topologists can't distinguish between a donut and a coffee mug: https://upload.wikimedia.org/wikipedia/commons/2/26/Mug_and_Torus_morph.gif).
- Exercises 3, 5

Section 23

- Read the definition of separation and connected.
- Read Lemma 23.1 and Examples 1 – 5.
- Skim the long list of useful theorems about connectedness. These will be useful if you choose to prove a space is connected.
- Exercises 1, 4, 5

Thursday, February 25

Work on your portfolio prior to class and come prepared to present. You could present a more refined (preferably typeset) version of a previous proof, or you could choose to work on something new. The following new properties are all related to connectedness (though not all of them appear in the text):

- connected
- locally connected
- totally disconnected
- path connected
- totally path disconnected
- locally path connected
- arc connected
- locally arc connected

- extremally disconnected
- biconnected
- hyperconnected
- ultraconnected

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, March 1

Section 26

- Read the definition of cover, compact, and the examples that follow.
- Read the statement and proof of Theorem 26.2. Notice the burden of proof to establish compactness is to show *every* (that is to say, a generic) open cover of the space contains a finite subcover.
- Read the statement and proof of Theorem 26.3. This theorem reinforces an idea we've seen before: It is often easier to prove Y^c is open rather than Y is closed directly. As you read the proof, sketch an picture that shows how x_0, y, Y, U_y, and V_y might appear in the plane. This is an excellent habit to have while reading complicated proofs.
- Exercises 2, 3

Section 28

- Read the definition of limit point compact. (This definition is actually missing from π-Base.)
- Read Theorem 28.1, it's proof, and the following examples. These are examples of limit point compact spaces that are not compact (so they would be good to consider if you wanted to add limit point compact to the database).
- Read the definition of sequentially compact. (This is probably the one you know from real analysis, but Theorem 28.2 shows all three notions of compactness are equivalent on the real line.)
- Exercises 2, 4

Thursday, March 3

Work on your portfolio prior to class and come prepared to present. You could present a more refined (preferably typeset) version of a previous proof, or you could choose to work on something new. The following new properties are all related to compactness (though not all of them appear in the text or even in π-Base):

- compact
- limit point compact
- sequentially compact
- countably compact
- quasicompact
- metacompact
- paracompact
- orthocompact
- Lindelöf
- Noetherian

- or put “locally” in front of basically any of these

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, March 15

Work on your portfolio prior to class and come prepared to present. You could present a more refined (preferably typeset) version of a previous proof, or you could choose to work on something new. The purpose of this extra presentation day is to make some definitive progress on your portfolio. Make it your goal to have at least three points worth of fully-typeset, completely-polished portfolio work by Thursday’s meeting.

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Thursday, March 17

Work on your portfolio prior to class and come prepared to present. Bring at least three points worth of fully-typeset, completely-polished portfolio work by Thursday’s meeting.

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, March 22

Section 30

- Read the definition of the first countability axiom and Example 1.
- Read the definition of the second countability axiom and the first two paragraphs of Example 3.
- Read Theorem 30.3 and its proof. (The first item defines a Lindelöf space, and the second item defines a separable space.)
- Example 4 introduces an important topology called the Sorgenfrey plane that we may encounter again.
- Exercise 1, 3, 4, 11, 12, 13, 14

Thursday, March 24

I will not be present for class today. Consider meeting anyway to solicit feedback from your peers regarding your portfolio. The following new properties are all related to countability:

- first-countable
- second-countable
- Lindelöf
- separable

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, March 29

Work on your portfolio prior to class and come prepared to present. You could present a more refined (preferably typeset) version of a previous proof, or you could choose to work on something new.

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Thursday, March 31

Work on your portfolio prior to class and come prepared to present. You could present a more refined (preferably typeset) version of a previous proof, or you could choose to work on something new.

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, April 5

Section 31

- Read the definitions of regular and normal. Figure 31.1 is a good way to visualize them.
- Read Lemma 31.1 and its proof. It provides conditions equivalent to regularity and normality in spaces that are T_1 or better.
- Read Examples 1 and 2 to get a better feel for regularity and normality. Example 3 shows that the product of normal spaces need not be normal, but you can see its proof is quite challenging.
- Exercise 1, 3, 7ab

Section 32

- Read Theorems 32.1 through 32.4 and their proofs. Each theorem relates normality to an older topological property. Note any parts of the proofs that seem unclear and ask about them during class.
- Exercise 1, 2, 4

Thursday, April 7

Work on your portfolio prior to class and come prepared to present. You could present a more refined (preferably typeset) version of a previous proof, or you could choose to work on something new. The following new properties are all related to regularity and normality (though not all of them appear in the text or even in π-Base):

- regular
- completely regular
- normal
- completely normal
- perfectly normal
- $T_{2\frac{1}{2}}$
- T_3
- T_4
- T_5
- T_6

Be sure to update the portfolio sheet accordingly.
https://docs.google.com/document/d/18VpLkruzK9425NGF1zH0x595SWzN2BL0fIV6TjvXQrc

Tuesday, April 12

Individual Meetings A

- 2:00 - Ann Marie
- 2:45 - Jordan

Come to my office (Olin 109D) to discuss your fifteen-minute Beamer presentation (see the course website for an example). Bring a handwritten outline of your talk that includes the main idea you want to convey as well as all the preliminary definitions required to understand that idea. Here are some example topics:

- Discuss the importance of a major topological theorem.
- Pick a single interesting space and explore a few familiar properties in that context.
- Pick a single interesting property and show how it behaves in a few familiar spaces.
- Discuss the interplay of a few related topological properties (e.g., the various T_i separation axioms).
- Present an important topological construction (e.g., the Stone-Čech Compactification) or concept (e.g., topological game theory).

Thursday, April 14

I will be out of town all day Thursday and after noon on Friday. Please schedule a time with me earlier in the week or on Friday morning.

Individual Meetings B

- Drew
- Thomas

Come to my office (Olin 109D) to discuss your fifteen-minute Beamer presentation (see the course website for an example). Bring a handwritten outline of your talk that includes the main idea you want to convey as well as all the preliminary definitions required to understand that idea. Here are some example topics:

- Discuss the importance of a major topological theorem.
- Pick a single interesting space and explore a few familiar properties in that context.
- Pick a single interesting property and show how it behaves in a few familiar spaces.
- Discuss the interplay of a few related topological properties (e.g., the various T_i separation axioms).
- Present an important topological construction (e.g., the Stone-Čech Compactification) or concept (e.g., topological game theory).

Tuesday, April 19

Individual Meetings A

- 2:00 - Ann Marie
- 2:45 - Jordan

Remember to work simultaneously on both your portfolio and your presentation. For this meeting, try to come with the following items:

- Introductory Beamer slides (title slide, preliminary definitions, etc.).
- A list of all the claims you plan to include in your final portfolio. You may simply update the portfolio organization document if you prefer.
- Proofs of the claims you intend to make in the presentation. These can be handwritten for now. Do not include them in the slides; we will discuss how much detail to include in the slides.

Thursday, April 21

Individual Meetings B

- 2:00 - Drew
- 2:45 - Thomas

Remember to work simultaneously on both your portfolio and your presentation. For this meeting, try to come with the following items:

- Introductory Beamer slides (title slide, preliminary definitions, etc.).
- A list of all the claims you plan to include in your final portfolio. You may simply update the portfolio organization document if you prefer.
- Proofs of the claims you intend to make in the presentation. These can be handwritten for now. Do not include them in the slides; we will discuss how much detail to include in the slides.

Tuesday, April 26

Individual Meetings A

- 2:00 - Ann Marie
- 2:45 - Jordan

Remember to work simultaneously on both your portfolio and your presentation. For this meeting, try to come with the following items:

- A finished (or nearly so) presentation.
- The proofs you intend to discuss in your presentation fully-detailed and typeset in LaTeX for your *portfolio*.

Remember that there is a big difference between the portfolio version and the presentation version of your proofs. The portfolio should be fully-rigorous with all the details we would expect out of a thorough textbook proof. Such detail is *not* appropriate for presentation slides, however. Instead, the text on your slides should help *guide* you and the listener through the main ideas in the proof.

Thursday, April 28

Individual Meetings B

- 2:00 - Drew
- 2:45 - Thomas

Remember to work simultaneously on both your portfolio and your presentation. For this meeting, try to come with the following items:

- A finished (or nearly so) presentation.
- The proofs you intend to discuss in your presentation fully-detailed and typeset in LaTeX for your *portfolio*.

Remember that there is a big difference between the portfolio version and the presentation version of your proofs. The portfolio should be fully-rigorous with all the details we would expect out of a thorough textbook proof. Such detail is *not* appropriate for presentation slides, however. Instead, the text on your slides should help *guide* you and the listener through the main ideas in the proof.

Tuesday, May 3

We'll meet at the usual time and place (Olin 110 at 2 pm) to practice our presentations. We'll need to move quickly to give everyone a chance to present and provide feedback, so please plan on getting started promptly.

Wednesday, May 4

Notice the nonstandard date and time.

The poster session starts at 9:00 am (location to be announced), so consider stopping by to support your peers.

The math presentation session starts at 10:20 am (location to be announced). Unless you have a very compelling reason, please plan on attending all the talks (there are six of them, at least one of which is your own). Make sure you email your updated talk to me before 9:00 am on Wednesday.

Friday, May 6

Deadline for portfolio rough drafts. Email a single pdf file containing all the proofs for your portfolio. I will have them available for pickup in my office by Monday.

Thursday, May 12

Final deadline for all assignments. Email the following to me by **8 am**:

- Updated responses to the reflection questions. (Do take a moment to see whether any of your thoughts have changed or been refined, but don't feel the need to give totally new responses for their own sake.)
- A single pdf file (and its associated tex file) containing all the proofs for your portfolio.
- Links to each of the pages in Pi-Base where you have uploaded the portfolio proofs. (If your claim already has an automatically deduced proof in place, you will not be able to upload it. In that case, indicate in your email that you could not upload it for this reason.)