

1. How many positive integers are there that are not larger than 1000 and are neither perfect squares nor perfect cubes?
2. Let  $F(n, k)$  be the number of partitions of the set  $[n]$  into exactly  $k$  blocks in which each block contains two or more elements. Express the numbers  $F(n, k)$  in terms of the Stirling numbers of the second kind.
3. Let  $G$  be the union of  $k$  disjoint cycles of length  $r$ . How many automorphisms does  $G$  have?
4. Let  $K_{n,n}$  be the simple graph whose vertex set consists of two  $n$ -element vertex sets  $A$  and  $B$ . Two vertices are adjacent in this graph if and only if one vertex belongs to  $A$  and the other to  $B$ . (In other words,  $K_{n,n}$  consists of all  $n^2$  edges *between*  $A$  and  $B$ , but no edges within  $A$  or within  $B$ ). How many *distinct* Hamiltonian cycles does  $K_{n,n}$  contain?

Take care that you do not overcount. For example, let  $K_{3,3}$  have vertex set  $A = \{1, 2, 3\}$  and  $B = \{a, b, c\}$ . The cycles  $1a2b3c1$ ,  $a2b3c1a$ , and  $1c3b2a1$  all contain the same edges, so they are the same cycle. The second cycle is a rotation of the first, and the third cycle is a reflection of the first. The cycle  $1b2a3c1$  is different than these, however, since it contains the edge (among others)  $1b$  while the others do not.