

1. Andy and Brenda play with dice. Andy will throw four dice at the same time. If at least one of the four dice shows a 6, then Andy wins. If not, then Brenda wins. Who has the greater chance of winning?
2. The National Football league consists of 32 teams. These teams are first divided into two conferences, the American Conference and the National Conference, each of which consists of sixteen teams. Each conference is further divided into four divisions of four teams each. Each division has a distinct name. In how many ways can this be done?
3. A magic square is a square matrix of nonnegative integers in which the sum of the entries in any row or column is equal. Let  $P_3(r)$  denote the number of  $3 \times 3$  magic squares that are symmetric about the main diagonal and have row and column sum equal to  $r$  (see example). Prove  $P_3(r) \leq (r + 1)^3$ .

$$\begin{array}{ccc} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 4 \end{array}$$

A  $3 \times 3$  magic square that is symmetric about its main diagonal and has row and column sum 4.

4. Let  $a_n$  be the number of compositions of  $n$  into parts that are 2 or larger. Describe a recurrence for  $a_n$  in terms of  $a_{n-1}$  and  $a_{n-2}$ .
5. Let  $F(n)$  be the number of set partitions having no singleton block. Construct a bijection to prove  $B(n) = F(n) + F(n + 1)$ .
6. Prove the inequality  $p(n)^2 < p(n^2 + 2n)$  for all positive integers  $n$ . (Hint: Read about Durfee squares on page 105, Exercise 8.)