

1. Select $n + 1$ distinct numbers from the set $\{1, 2, 3, \dots, 2n\}$. Prove there will always be two among the selected integers whose greatest common divisor is 1.
2. Place 33 points inside a regular triangle of side length 1. Prove there will always be three points among them such that the triangle they define has area less than 0.03.
3. Let $a_1 = 5$ and $a_{n+1} = a_n^2$. Prove that the last n digits of a_n are the same as the last n digits of a_{n+1} . (Hint: This is equivalent to proving $a_{n+1} - a_n$ is divisible by 10^n .)
4. Let $a_0 = a_1 = 1$, and let $a_n = a_{n-1} + 5a_{n-2}$ for $n \geq 2$. Prove that $a_n \leq 3^n$ for all $n \geq 0$.